BLOW-UP SYLLABUS
ENGINEERING PHYSICS (18PHY12/22)
(Common to all Branches)
(Effective from the academic year 2018-19)
MODULE – 1

Sl. No	Details	Duration	Remarks
1	**1.1 Free Oscillations:** Definition of SHM, Characteristics, Examples and Derivation of differential equation of motion for SHM starting from Hookes' law $\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$ and mention its solution	1/2hr	No numerical problems
2	Mechanical simple harmonic oscillator: Mass suspended to spring (vertical vibrations) - Description, Mention of Expression for time period/frequency, Definition of force constant and its significance, Derivation of expressions for force constants for series and parallel combination of springs.($k_s = \frac{k_1 k_2}{k_1 + k_2}$ and $k_p = k_1 + k_2$) Complex notation of simple harmonic motion ($Ae^{i(\omega t + \varepsilon)}$), Phasor representation of simple harmonic motion	1and 1/2hr	Numerical problems on T,f and k
3	Definition of free oscillations with examples, mention the equation of motion, Natural frequency of vibration – Qualitative discussion.	1/2hr	Numerical problems on natural frequency
4	**1.2 Damped oscillations:** Definition with examples. Derivation of decaying amplitude, Discussion of 3 cases viz, over damping, critical damping and underdamping. Quality factor: Definition, equation and its significance,	1and 1/2hr	Numerical problems on damping and quality factor
5	**1.3 Forced oscillations:** Definition with examples. Derivation of expressions for amplitude and phase of forced vibrations	1and 1/2hr	Numerical problems
	Discussion of 3 cases (i) p<<ω, (ii) p= ω and (iii) p>> ω		
	Resonance: Definition, Examples, Condition for resonance and expression for maximum amplitude (just mention) .		
	Sharpness of Resonance: Definition and significance, mention the effect of damping on sharpness of resonance		
	Qualitative discussion of Examples of Resonance: Helmholtz Resonator- Description and mention of expression for resonant frequency		No numerical problems
6	**1.4 SHOCK WAVES:** Definition of Mach number, classification of objects based on Mach number (subsonic, supersonic, Transonic and hypersonic) Definition and properties of shock waves	1hr	Numerical problems on Mach number
7	Definition of control volume, Laws of conservation of mass, energy and momentum (Statement and equations)	1 and ½ hr	No numerical problems
8	Construction and working of Reddy shock tube Applications of shock waves: Qualitative (minimum 5 applications)		No numerical problems
9	Tutorial classes	2hr	Involvement of students in respect of their doubts about the module and numerical problems

Sl. No	Details	Duration	Remarks
1	**2.1 Elasticity:** Explain elasticity and plasticity. Give some examples for good elastic materials. Mention the importance (Engineering) of elastic materials. concept of stress and strain. Discuss two types of stresses namely tensile stress and compressive stress. Briefly discuss the effect of stress, temperature, annealing and impurities on elasticity	1 and ½ hr	No numerical problems
2	Strain hardening and softening: just explain what is strain hardening (strengthening of material by plastic deformation) and hardening co efficient and softening. No detailed discussion of processes.	1/2hr	No numerical problems
3	State and explain Hookes' law, stress strain curve, elastic and plastic limits. Elastic modulus, define three different elastic moduli. Write equations for each moduli like $Y = \frac{FL}{A\Delta L}$ & so on.	1/2hr	Numerical problems on Y, η and K
4	**2.2 Poisson's ratio:** Define lateral strain and linear strain and hence Poisson's ratio σ=β/α (α= linear strain coefficient) and (β= lateral strain coefficient)	1hr	Numerical problems
5	Relation between shear strain, longitudinal and compression strain. Show that longitudinal strain + compression strain = shear strain by considering a cubical elastic body		No numerical problems
6	Derive the relation between Y, η and σ Derive the relation between K, Y and σ		Numerical problems
7	Derive the relation between K, η and Y	1 and 1/2hr	
8	Discuss the limiting values of σ and limitations of Poisson's ratio		No numerical problems
9	**2.3 Bending of beams:** Definition of beams, different types of beams and mention their Engineering applications. Definition of neutral surface/plane and neutral axis.	1/2hr	No numerical problems
10	Define bending moment. Derive the expression for bending moment in terms of moment of inertia ($BM = \frac{Y}{R}I_g$)	1 hr	No numerical problems
11	Mention the expression for bending moment for circular and rectangular cross sections		Numerical problems
12	Describe a single cantilever and hence derive the expression for Y (for rectangular beam) (only depression)	½ hr	Numerical problems
14	**2.4 Torsion of a cylinder:** Twisting couple on cylindrical wire, explain torsional oscillations, derive the expression for couple per unit twist for a solid cylinder	1 hr	Numerical problems
15	Mention the expression for Time period of torsional oscillations $T = 2\pi\sqrt{I_g/C}$. Brief explanation of applications of torsional pendulum		Numerical problems
16	Tutorial classes	2hr	Involvement of students in respect of their doubts about the module and numerical problems

Sl No	Details	Duration	Remarks
1	**Only Cartesian co ordinates must be used in both theory and problems** **3.1 Maxwell's equations:** Fundamentals of vector calculus: Briefly explain scalar product, vector product, ∇ operation, concept of divergence, gradient and curl along with physical significance and examples like Div and curl of E and B	1 and ½ hr	Numerical problems of div and curl
2	Discuss the three different types of integrations *viz* linear, surface and volume integrations. Derivation of Gauss divergence theorem, mention Stokes' theorem		No numerical problems
3	Explain briefly Gauss flux theorem in electrostatics and magnetism, Ampere's law, Biot-Savart's law and Faraday's laws of electromagnetic induction	½ hr	Numerical problems
4	Discuss continuity equation, definition of displacement current(I_d), expression for displacement current, Maxwell-Ampere's law	½ hr	Numerical problems on (I_d)
5	List of four Maxwell's equations in differential form and in vacuum	½ hr	No numerical problems
6	**3.2 EM Waves:** Derive wave equation in terms of electric field using Maxwell's equations. Mention of plane electromagnetic waves in vacuum along with the equations for E, B and *c* in terms of μ_0 and ε_0 and E and B	1 and ½ hr	Numerical problems on calculation of 'c' and on equations of E and B
7	Explain the transverse nature of electromagnetic waves, three types of polarization namely linear, elliptical and circular polarization of E.		Numerical problems
8	**3.3 Optical fiber:** Description of propagation mechanism of light through an optical fiber. Angle of acceptance and numerical aperture(NA): Theory with condition for propagation	1 and ½ hr	Numerical problems on θ_C Numerical problems on angle of acceptance, NA, V number, modes of propagation
9	Modes of propagation and V number and types of optical fibers(qualitative)		
10	Attenuation: Definition of attenuation, name the three types of attenuation, Causes of attenuation: Explain absorption, scattering and radiation losses. Mention the expression for attenuation coefficient	2hr	Numerical problems on attenuation coefficient
11	Application of optical fiber: Point to point communication: Explain with the help of block diagram. Merits and de merits of optical fiber communication.		No numerical problems
12	Tutorial classes	2hr	Involvement of students in respect of their doubts about the module and numerical problems

MODULE-4

Sl No	Details	Duration	Remarks
1	**4.1 Quantum Mechanics:** Introduction to need of Quantum mechanics with a discussion of Planck's equation for energy density	½ hr	No numerical problems
2	Wave nature of particles–De Broglie hypothesis followed by wavelength equations, extended to accelerated electron	½ hr	Numerical problems
3	Heisenberg's uncertainty principle-Statement and mention the three uncertainty relations. Applications of uncertainty principle- to show the non confinement of electrons in the nucleus (by considering diameter of nucleus). Energy relativistic equation shall not be considered.	1 hr	Numerical problems
4	Schrodinger's time independent wave equation –Setting up of Schrodinger's time independent wave equation using $\psi=Ae^{i(kx-wt)}$.	1 hr	No numerical problems.
5	Significance of Wave function –qualitative statement regarding wave function, Probability density, Max born interpretation, Normalization, and Properties of wave function		No Numerical problems
6	Application Schrodinger's wave equation to particle in 1-D potential well of infinite height and obtain the energy Eigen values and eigen functions. Probability densities	1hr	Numerical problems
7	**4.2 Laser:** Brief discussion of spontaneous and stimulated processes – Explanation of the process of induced absorption, spontaneous and stimulated emission.	½ hr	No numerical problems
8	Einstein's coefficients (expression for energy density) – derivation of energy density in terms of Einstein's co efficients	1 hr	Numerical problems
9	Requisites of a Laser system – a brief explanation about active medium, resonant cavity and exciting system.		No numerical problems
10	Conditions for laser action-To explain population inversion and meta stable state		Numerical problems
13	Principle: mention different modes of vibrations of CO_2, explain construction and working of CO_2 laser with energy level diagram experimental setup.	2 hr	No numerical problems
14	Principle, Construction and working of semiconductor Lasers – Explain principle, construction and working of homo junction semiconductor laser with energy level diagram and experimental setup.		numerical problems
15	Application of Lasers in Defense (Laser range finder) – qualitative explanation about application of laser as laser range finder.	1/2hr	No numerical problems
16	Application of Lasers in Engineering (Data storage) - qualitative explanation about application of laser in data storage (compact disc, DVD).		No numerical problems
17	Tutorial classes	2 hrs	Involvement of students in respect of their doubts about the module and numerical problems

Sl. No	Details	Duration	Remarks
1	**5.1 Quantum free electron theory:** Review of classical free electron theory (just mention who proposed it and what for it was proposed), mention the expressions for electrical conductivity based on classical free electron theory, and explain the failures of classical free electron theory (in terms of relation between conductivity and temperature, and relation between conductivity and free electron density, with specific examples)	½ hr	No numerical problems
2	Assumptions of quantum free electron theory, definition of density of states and mention the expression for density of states (No derivation)	1 and 1/2hr	Numerical problems on density of states, Fermi energy, Fermi factor
3	Qualitative discussion of Fermi level, Fermi energy, Fermi-Dirac statistics, Fermi factor, Fermi factor at different temperatures (3 cases).		
4	Derivation of the expression for Fermi energy at zero Kelvin. Mention the expression Fermi velocity and Fermi temperature. Expression for electrical conductivity in terms of Fermi velocity, mean free path and effective mass (No derivation).	½ hr	Numerical problems on Fermi velocity, conductivity
5	Success of quantum free electron theory (in terms of relation between conductivity and temperature, and relation between conductivity and free electron density, with specific examples)	½ hr	No numerical problems
6	**5.2 Semiconductors:** Fundamentals of semiconductor. Description of Fermi level in intrinsic semiconductor. Mention of expression for electron and hole concentration in intrinsic semiconductors. Derivation of relation between Fermi energy and energy gap for an intrinsic semiconductor.	1hr	No numerical problems
7	Derivation of the expression for electrical conductivity of semiconductors, Explanation of Hall effect with Hall voltage and Hall field, derivation of the expression for Hall coefficient.	1 hr	Numerical problems on conductivity, Hall effect
8	**5.3 Dielectrics:** Fundamentals of dielectrics. Polarisation, mention the relation between dielectric constant and polarization. Types of polarization. Polar and non-polar dielectrics	1 hr	No numerical problems
9	Definition of internal field in case of solids and mention of its expression for one dimensional case. Mention the expressions for internal field for three dimensional cases and Lorentz field. Derivation of Clausius-Mossotti equation.	1 hr	Numerical problems on internal field and Clausius-Mossotti equation
10	Description of solid, liquid and gaseous dielectrics with one example each. Qualitative explanation of applications of dielectrics in transformers.	1/2hr	No numerical problems
11	Tutorial classes	2hr	Involvement of students in respect of their doubts about the module and numerical problems

Text Books:
1. A Text book of Engineering Physics- M.N. Avadhanulu and P.G. Kshirsagar, 10th revised Ed, S. Chand & Company Ltd, New Delhi
2. Engineering Physics-Gaur and Gupta-Dhanpat Rai Publications-2017
3. Concepts of Modern Physics-Arthur Beiser: 6th Ed;Tata McGraw Hill Edu Pvt Ltd- New Delhi 2006

Reference books:
1. Introduction to Mechanics — MK Verma: 2nd Ed, University Press(India) Pvt Ltd, Hyderabad 2009
2. Lasers and Non Linear Optics – BB laud, 3rd Ed, New Age International Publishers 2011
3. Solid State Physics-S O Pillai, 8th Ed- New Age International Publishers-2018
4. Shock waves made simple- Chintoo S Kumar, K Takayama and KPJ Reddy: Willey India Pvt. Ltd. New Delhi2014
5. Introduction to Electrodynamics- David Griffiths: 4th Ed, Cambridge University Press 2017

Module wise text books/Reference Books

Module	Article No	Text Book/Reference Book
I	1.1	1. Engineering Physics-Gaur and Gupta-Dhanpat Rai Publications-2017
	1.2	
	1.3	
	1.4	1. Shock waves made simple- Chintoo S Kumar, K Takayama and KPJ Reddy: Willey India Pvt. Ltd. New Delhi2014
II	2.1	1. Engineering Physics-Gaur and Gupta-Dhanpat Rai Publications-2017
	2.2	2. Introduction to Mechanics — MK Verma: 2nd Ed, University Press(India) Pvt Ltd, Hyderabad 2009
	2.3	
	2.4	
III	3.1	1. A Text book of Engineering Physics- M.N. Avadhanulu and P.G. Kshirsagar, 10th revised Ed, S. Chand & Company Ltd, New Delhi
	3.2	2. Introduction to Electrodynamics- David Griffiths: 4th Ed, Cambridge University Press 2017
	3.3	
IV	4.1	1. A Text book of Engineering Physics- M.N. Avadhanulu and P.G. Kshirsagar, 10th revised Ed, S. Chand & Company Ltd, New Delhi 2. Concepts of Modern Physics-Arthur Beiser: 6th Ed;Tata McGraw Hill Edu Pvt Ltd- New Delhi 2006
	4.2	1. Lasers and Non Linear Optics – BB laud, 3rd Ed, New Age International Publishers 2011
V	5.1	1. Concepts of Modern Physics-Arthur Beiser: 6th Ed;Tata McGraw Hill Edu Pvt Ltd- New Delhi 2006
	5.2	2. Solid State Physics-S O Pillai, 8th Ed- New Age International Publishers-2018
	5.3	1. A Text book of Engineering Physics- M.N. Avadhanulu and P.G. Kshirsagar, 10th revised Ed, S. Chand & Company Ltd, New Delhi

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Module-1
Oscillations and Waves & Shock Waves

Blowup of the syllabus: (RBT Levels L1, L2, L3)
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Free Oscillations: Definition of SHM, derivation of equation for SHM, Mechanical and electrical simple harmonic oscillators (mass suspended to spring oscillator), complex notation and phasor representation of simple harmonic motion. Equation of motion for free oscillations, Natural frequency of oscillations.

Damped and forced oscillations: Theory of damped oscillations: over damping, critical & under damping, quality factor. Theory of forced oscillations and resonance, Sharpness of resonance. One example for mechanical resonance.

Shock waves: Mach number, Properties of Shock waves, control volume. Laws of conservation of mass, energy and momentum. Construction and working of Reddy shock tube, applications of shock waves. Numerical problems

1.1 FREE OSCILLATIONS

Oscillations and vibrations play a more significant role in our lives than we realize. When you strike a bell, the metal vibrates, creating a sound wave. All musical instruments are based on some method to force the air around the instrument to oscillate. Oscillations from the swing of a pendulum in a clock to the vibrations of a quartz crystal are used as timing devices. When you heat a substance, some of the energy you supply goes into oscillations of the atoms. Most forms of wave motion involve the oscillatory motion of the substance through which the wave is moving. Despite the enormous variety of systems that oscillate, they have many features in common with the simple system of a mass on a spring. The harmonic oscillators have close analogy in many other fields; mechanical example of a weight on a spring, oscillations of charge flowing back and forth in an electrical circuit, vibrations of a tuning fork, vibrations of electrons in an atom generating light waves, oscillation of electrons in an antenna etc.,

SIMPLE HARMONIC MOTION

A mass is said to be performing Simple Harmonic Motion when the mass is the restoring force is proportional to the displacement. The restoring force is directed opposite to displacement.

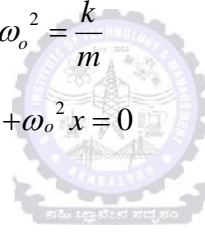Restoring force α – displacement
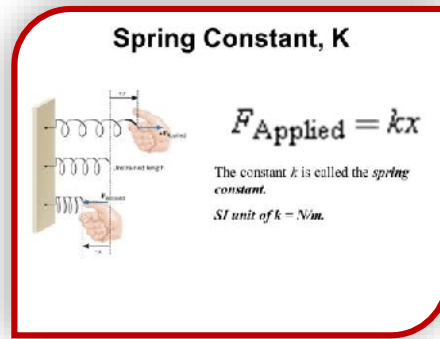
$$F = -k\ x$$

Here k is the proportionality constant known as spring constant. It represents the amount of restoring force produced per unit elongation and is a relative measure of stiffness of the material.

$$F_{\text{Re}\,storing} = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$Let\ \omega_o^{\,2} = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \omega_o^{\,2}x = 0$$

Here ω_o is angular velocity = 2.π.f

f is the natural frequency $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

The Solution is of the form   x(t) = A cosω$_o$t + B sinω$_o$t.

This can also be expressed as x(t) = C cos(ω$_o$t-φ) where $C = \sqrt{A^2 + B^2}$ tanφ = B/A

Mechanical Simple Harmonic Oscillator:

We consider a mechanical spring which resists compression / elongation to be elastic. At the lower end of the spring, a body of mass m is attached. Mass of the spring is neglected .When the body is pulled down by a certain distance x and then released, it undergoes SHM. **When there are no external forces, the oscillations are said to be free oscillations.** The mass oscillates with its natural frequency.

 The motion of a mass m attached to a spring follows a linear differential equation.

$$F_{\text{Re}\,storing} = -kx$$

From Newton's second law, the equation of motion is written as

$$\frac{d^2x}{dt^2} + kx = 0$$

This is a second order homogeneous linear differential equation.

Auxiliary equation is $\left(D^2 + \omega^2\right)x = 0$

Roots are $D = +i\omega$ and $D = -i\omega$

The general solution is
$$
\begin{aligned}
x &= Ae^{i\omega t} + Be^{-i\omega t} \\
&= A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t) \\
&= (A+B)\cos\omega t + i(A-B)\sin\omega t \\
&= C\cos\omega t + D\sin\omega t
\end{aligned}
$$

This may also be expressed as $x = A\cos(\omega t - \phi)$

where $A = \sqrt{C^2 + D^2}$ and $\phi = \tan^{-1}(D/C)$
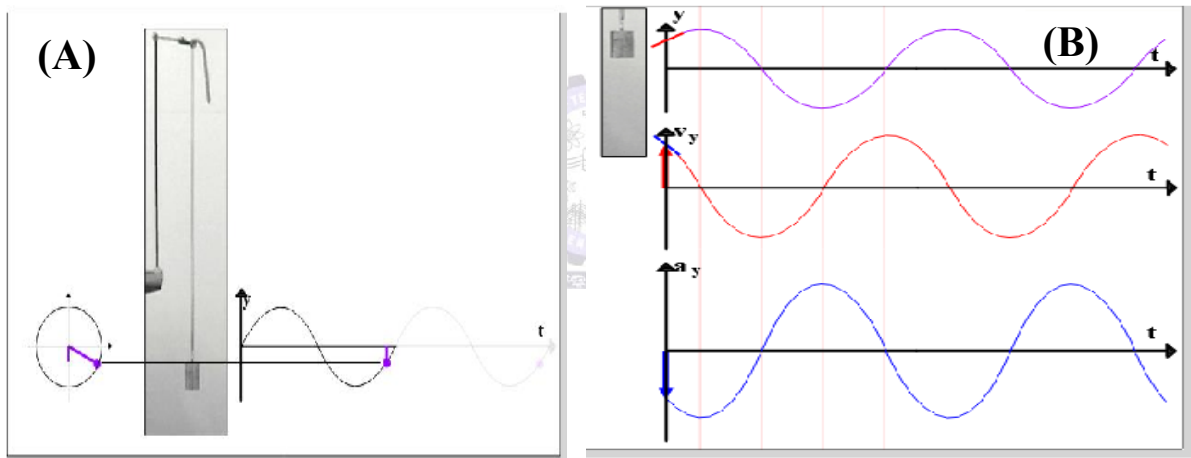
Fig. 1.1.(A) SHM as a projection of uniform circular motion and (B) Displacement, velocity and acceleration graphs for SHM.

{Reference: http://www.animations.physics.unsw.edu.au/jw/SHM.htm}

Velocity of particle $V_{Max} = \dfrac{dx}{dt} = -\omega A$

Acceleration of particle $= \dfrac{dv}{dt} = -\omega^2 x$

$$angular\ velocity\ \omega = \sqrt{\frac{k}{m}}$$

$$Period = 2\pi\sqrt{\frac{m}{k}}$$

$$Phase = \omega t$$

$$natural\ frequency = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Complex Notation

Complex numbers are a convenient tool to mathematically analyze sinusoidal functions. It can be used to represent amplitude and phase of a periodically varying function.

$$Rectangular\ form : z = x + jy$$

$$Polar\ form\ :\ z = r\angle\theta$$

$$Exponential\ form : z = re^{j\phi}$$

Phasors are Time Independent complex quantities used to represent periodically varying parameters.

Ex- Alternating current is represented as $I(t) = \hat{I}e^{i\omega t}$

Alternating voltage is represented as $V(t) = \hat{V}e^{i\omega t}$

Here $\hat{I}$ and $\hat{V}$ are phasors

A periodically force is expressed in phasor form as

$F = \hat{F}\cos\omega t = \mathrm{Re}al\ part\ of\ F_o(\cos\omega t + i\sin\omega t)$. Here $\hat{F}$ is a phasor

Ex: Mechanical

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = \frac{F_o}{m}\cos\omega t$$

$$\frac{d^2(x_r + ix_i)}{dt^2} + \frac{k(x_r + ix_i)}{m} = \frac{F_r + iF_i}{m}$$

Electrical: Phasor representation of Impedance in LCR circuit
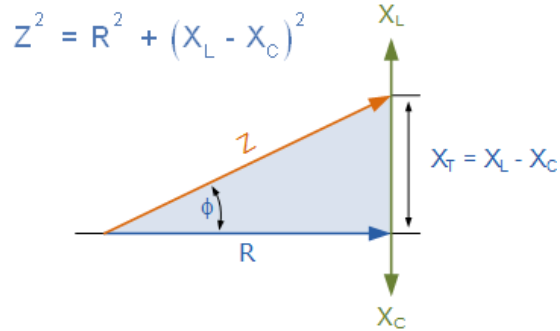
$$z = R + j(\omega L - \frac{1}{\omega C})$$

$$Z^2 = R^2 + (X_L - X_C)^2$$

X_L

Z

$X_T = X_L - X_C$

ϕ

R

X_C

Fig.1.2. Phasor representation of impedance in LCR circuit.

Expression for Spring Constant for Series Combination
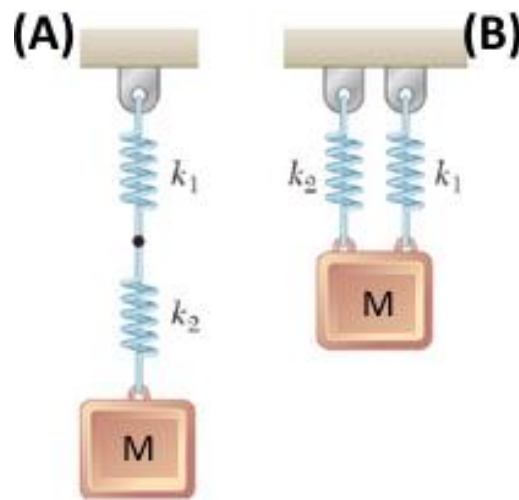
(A) k_1 k_2 M

(B) k_2 k_1 M

Fig.1.3. (A) Series combination and (B) Parallel combination of two springs having spring constant k_1 and k_2.

Consider a load suspended through two springs with spring constants k_1 and k_2 in series combination as shown in Fig.1.3 (A). Both the springs experience same stretching force. Let Δx_1 and Δx_2 be their elongation.

Total elongation   is given by

$$\Delta X = \Delta X_1 + \Delta X_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{F}{k_{eqv}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eqv}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Expression for Spring Constant for Parallel Combination

Consider a load suspended through two springs with spring constants k_1 and k_2 in parallel combination as Fig.1.3 (B). The two individual springs both elongate by x but experience the load non-uniformly. Total load across the two springs is given by

$$F = F_1 + F_2$$
$$k_{eqv}.\Delta X = k_1.\Delta X + k_2.\Delta X$$
$$k_{eqv} = k_1 + k_2$$

Free Oscillations

The oscillations are said to be free oscillations when there are no external forces. The object oscillates with natural frequency.
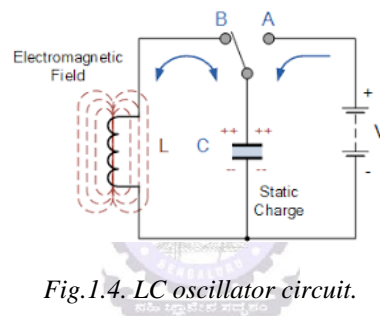
Ex: **ELECTRICAL OSCILLATIONS- LC Oscillations**

Fig.1.4. LC oscillator circuit.

If the capacitor is initially charged and the switch is then closed, we find that the charge on the capacitor oscillates.

Voltage across capacitor = q/C

Voltage across Inductor = $L\dfrac{d^2q}{dt^2}$

For the above circuit, $q/C + L\dfrac{d^2q}{dt^2} = 0$

$$\frac{d^2q}{dt^2} + \frac{q}{LC} = 0$$

Comparing this with $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$;   the solution may be written as q = A cosωt + B sinωt where

$$\omega = \frac{1}{\sqrt{LC}}$$

SIMILARITIES BETWEEN MECHANICAL OSCILLATIONS AND ELECTRICAL OSCILLATIONS

Variable	Mechanical Property	Electrical Property
Independent Variable	Time	Time
Dependent variable	Position(x)	Charge(q)
Inertia	mass	Inductance
Resistance	Drag coefficient	Resistance (R/L)
Stiffness	k	1/C
Resonant frequency	$\omega_o = \sqrt{k/m}$	$\omega_o = \dfrac{1}{\sqrt{LC}}$
Energy	Potential energy $= \dfrac{1}{2}kx^2$ Kinetic energy $= \dfrac{1}{2}mv^2$	$\dfrac{1}{2}CV^2$ $\dfrac{1}{2}LI^2$

Damped Oscillations

Mechanical Case:

In a damped harmonic oscillator, the amplitude decreases gradually due to losses such as friction, impedance etc. The oscillations of a mass kept in water, charge oscillations in a LCR circuit are examples of damped oscillations. Let us assume that in addition to the elastic force F = -kx, there is a force that is opposed to the velocity, F = b v where b is a constant known as resistive coefficient and it depends on the medium, shape of the body.
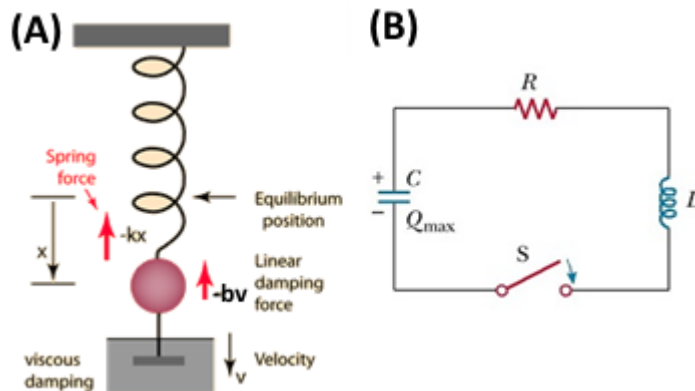

Fig.1.5. (A) Spring oscillation under damping created by viscous liquid. (B) Equivalent LCR circuit in series

For the oscillating mass in a medium with resistive coefficient b, the equation of motion is given by

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

This is a homogeneous, linear differential equation of second order.

The auxiliary equation is $\quad D^2 + \dfrac{b}{m}D + \dfrac{k}{m} = 0$

The roots are $D_1 = -\dfrac{b}{2m} + \dfrac{1}{2m}\sqrt{b^2 - 4mk}$ and $D_2 = -\dfrac{b}{2m} - \dfrac{1}{2m}\sqrt{b^2 - 4mk}$

The solution can be derived as $\quad x(t) = Ce^{-\left(\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}\right)t} + De^{-\left(\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2 - 4mk}\right)t}$

This can be expressed as x(t) = $Ae^{-\frac{b}{2m}t}\cos(\omega t - \phi)$ where $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2}$

$$A = \sqrt{C^2 + D^2} \qquad \phi = \tan^{-1}(D/C)$$

Here, the term $Ae^{-\frac{b}{2m}t}$ represents the decreasing amplitude and (ωt-ϕ) represents phase.

Case 1: $\quad b^2 > 4mk \qquad$ OVER DAMPING

Case 1: $\quad b^2 < 4mk \qquad$ UNDER DAMPING

Case 1: $\quad b^2 = 4mk \qquad$ CRITICAL DAMPING

Under damped: $\quad b^2 > 4mk$

　　　When the retarding force is less than k. A, the system oscillates with decreasing amplitude

Critically damped: $\quad b^2 = 4mk$

　　　When $\dfrac{b}{2m} = \omega_o$, the system does not oscillate

Over damped: $\quad b^2 > 4mk$

　　　When the retarding force is greater than k.A, $\dfrac{b}{2m} > \omega_o$

Over damping takes away the energy of the system and oscillations stop.

Electrical case:

For the above circuit, $\quad \dfrac{q}{c} + R\dfrac{dq}{dt} + L\dfrac{d^2q}{dt^2} = 0$
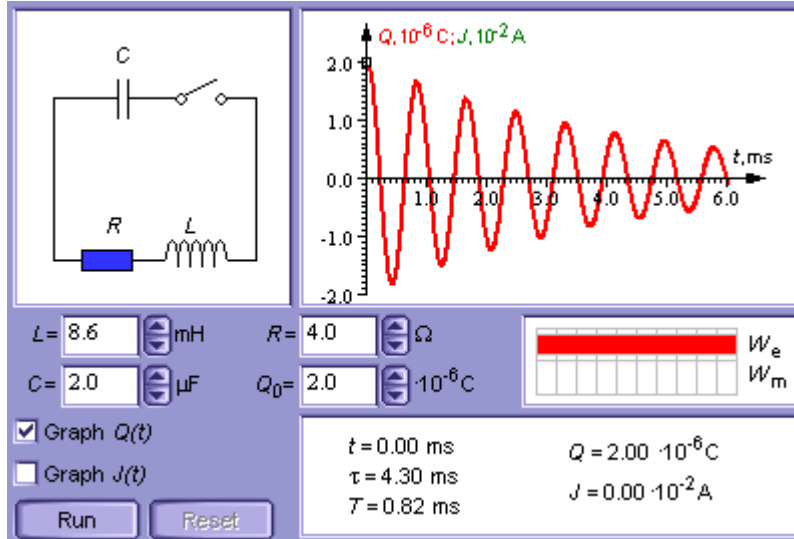

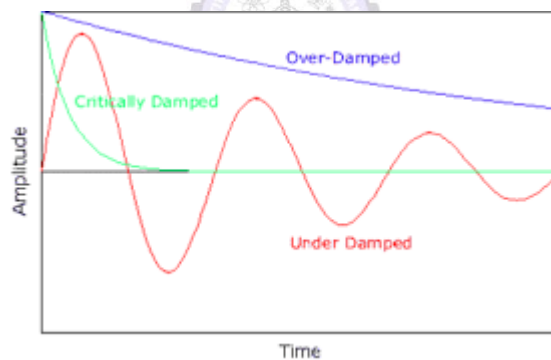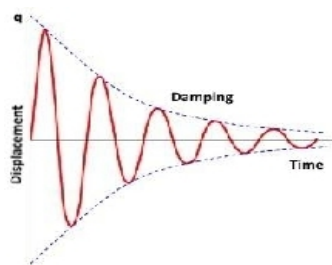*Fig.1.6. Damped Oscillations of LCR circuit as measured.*

Fig. 1.7. Various kinds of damping oscillations and their die out with respect to time.

The solution is of the form $q = q_o e^{-\frac{Rt}{2L}}\cos\omega t \qquad$ where $\omega = \left[\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2\right]^{\frac{1}{2}}$

FORCED OSCILLATIONS

Mechanical:

Oscillations that result when an external oscillating force is applied to the particle subject to SHM. For example, a child on a swing can be kept in motion by appropriately timed "pushes." The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the decrease in mechanical energy in each cycle that results from resistive forces. Vibrations of tuning fork placed on a resonating box make the walls of the box and the air inside oscillate.

Let $F = F_o \cos\omega_f t$ be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx + bv + F_o \cos\omega_f t$$

$$m\frac{d^2x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \cos\omega_f t \qquad \text{.......(1)}$$

This equation is non-homogeneous. The complimentary function is given by

$$x_c(t) = C \cos\omega_f t + D\sin\omega_f t \qquad \text{.......(2)}$$

This can also be expressed as $x_c(t) = A \cos(\omega_o t - \varphi)$ where $A = \sqrt{C^2 + D^2}$ $\tan\varphi = D/C$ ......(3)

$$x_c^\bullet(t) = -C\omega \sin\omega_f t + D\omega \cos\omega_f t$$

$$x_c^{\bullet\bullet}(t) = -C\omega^2 \cos\omega_f t - D\omega^2 \sin\omega_f t$$

Substituting in (1)

$$\left[(k - m\omega_f^2)C + Db\omega_f\right]\cos\omega_f t + \left[-bC\omega_f + \left(k - m\omega_f^2\right)D\right]\sin\omega_f t = F_o \cos\omega_f t$$

By equating coefficients of the sine and cosine terms on both sides

$$\left(k - m\omega_f^2\right)a + Db\omega_f = F_o$$

$$-bC\omega_f + \left(k - m\omega_f^2\right)d = 0$$

Solving for c and d

$$C = F_o \frac{k - m\omega_f^2}{\left(k - m\omega_f^2\right)^2 + b^2\omega_f^2}$$

$$D = F_o \frac{b\omega_f}{\left(k - m\omega_f^2\right)^2 + b^2\omega_f^2}$$

Substituting for $\omega_o = \sqrt{k/m}$

$$C = F_o \frac{m(\omega_o^2 - \omega_f^2)}{m^2\left(\omega_o^2 - \omega_f^2\right)^2 + b^2\omega_f^2} \qquad\qquad D = F_o \frac{b\omega_f}{m^2\left(\omega_o^2 - \omega_f^2\right)^2 + b^2\omega_f^2}$$

The general solution is

$$x(t) = F_o \frac{m(\omega_o^2 - \omega_f^2)}{m^2\left(\omega_o^2 - \omega_f^2\right)^2 + b^2\omega_f^2}\cos\omega_f t + F_o \frac{b\omega_f}{m^2\left(\omega_o^2 - \omega_f^2\right)^2 + b^2\omega_f^2}\sin\omega_f t$$

Using (3), the solution can also be expressed as

$$x(t) = A\cos(\omega_o t - \varphi)$$

where amplitude $A = \sqrt{C^2 + D^2}$ and phase $\varphi = \tan^{-1}D/C = \tan^{-1}\left(\dfrac{b\omega_f}{m(\omega_o^2 - \omega_f^2)}\right)$

(**Note:** Refer an alternative method in APPENDIX)

Case 1: $\omega_f < \omega_o$, $\qquad A \approx \dfrac{F_0}{k\left(1 - \left(\dfrac{\omega_f}{\omega_0}\right)^2\right)} \Rightarrow A = F_o/k$ - Spring constant controls the oscillations

Case 2: $\omega_f = \omega_o$, $\quad b = 0$ (undamped) $\quad A \Rightarrow \infty$: **RESONANCE CONDITION**

When the frequency of the applied force is same as natural frequency of the oscillator, resonance occurs. Maximum transfer of energy takes place.

Case 3: $\omega_f > \omega_o$, $\qquad A \approx \dfrac{F_0}{m\omega^2\cos\omega t}$ Driving force controls the oscillations

Note: Phase lag between the applied force and displacement increases from 0 to π/2 as driving frequency increases from 0 to ω_o
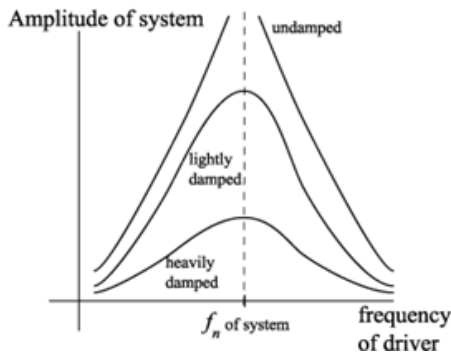
Forced oscillations : Electrical:

$$\frac{q}{c} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} = V_o e^{iwt} = \widehat{V}$$

$$\widehat{q} = q_o e^{iw\,t}$$

$$\left[L(iw)^2 + R(iw) + \frac{1}{c}\right]q_o e^{iw\,t} = V_o e^{iw\,t}$$

$$\widehat{q} = \frac{\widehat{V}}{L\left(i\omega^2 - R(i\omega) + \frac{1}{C}\right)}$$

$$\widehat{I} = \frac{d\widehat{q}}{dt} = iw\widehat{q}$$

$$\widehat{V} = [R + i(wL - \frac{1}{wC})]\widehat{I} = \widehat{I}\widehat{Z}$$

Electrical Mechanical Analogy
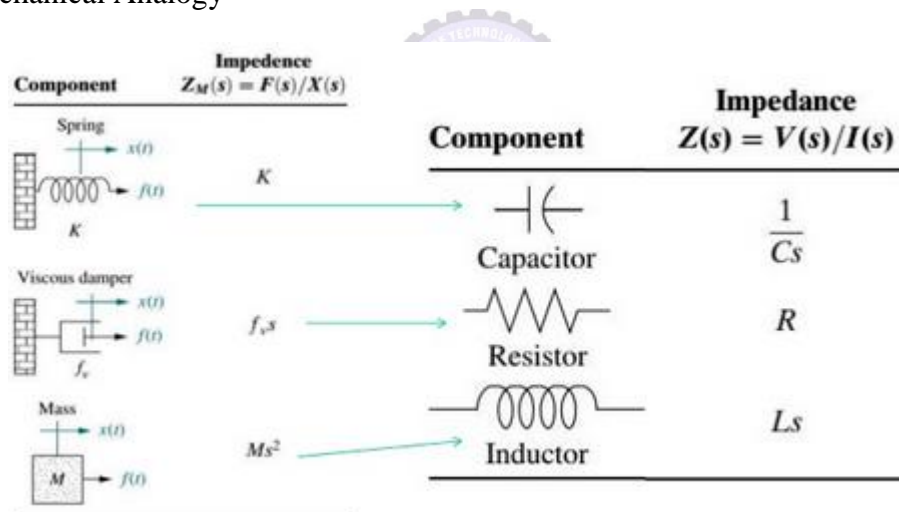
FORCED OSCILLATIONS –Alternative method

Let F = F$_o$ Cosω$_f$t be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx + bv + F_o \sin \omega_f t$$

$$m\frac{d^2 x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \sin \omega_f t$$

$$\frac{d^2 x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_o}{m} \sin \omega_f t$$

$$Let \quad \frac{b}{m} = 2k; \frac{k}{m} = \omega^2; \frac{F_o}{m} = F$$

$$\frac{d^2 x}{dt^2} + 2k\frac{dx}{dt} + \omega_o{}^2 x = F \sin \omega_f t \qquad\qquad …..(1)$$
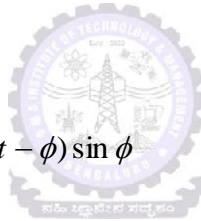
Let one particular solution be $y = a.\sin(\omega_f t - \phi)$

$$\frac{dy}{dt} = \omega_f a.\cos(\omega_f t - \phi)$$

$$\frac{d^2 y}{dt^2} = -\omega_f^2 a.\sin(\omega_f t - \phi)$$

Also

$$F \sin \omega_f t = F.\sin(\omega_f t - \phi + \phi)$$
$$= F \sin(\omega_f t - \phi)\cos\phi + F \cos(\omega_f t - \phi)\sin\phi$$

Substituting in (1)

$$-\omega_f^2 a.\sin(\omega_f t - \phi) + 2ka\omega_f \cos(\omega_f t - \phi) + \omega_o^2 a \sin(\omega_f t - \phi) = F \sin(\omega_f t - \phi)\cos\phi + F \cos(\omega_f t - \phi)\sin\phi$$

Comparing coefficients of $\sin(\omega_f t - \phi)$ and $\cos(\omega_f t - \phi)$ on both sides

$$a(.\omega_o^2 - \omega_f^2) = F \cos\phi$$
$$2ka\omega_f = F \sin\phi$$

$$\therefore F^2 = a^2(\omega_o^2 - \omega_f^2)^2 + 4k^2 a^2 \omega_f^2$$

$$a = \frac{F}{\sqrt{(\omega_o^2 - \omega_f^2)^2 + 4k^2\omega_f^2}} \qquad\qquad \tan\phi = \frac{2k\omega_f}{\omega_o^2 - \omega_f^2}$$

EXAMPLES OF RESONANCE

1. Oscillations of the stretched string kept under the influence of oscillating magnetic field caused by oscillating current. The string vibrates with maximum amplitude when the applied frequency matches with Natural frequency of the string.

2. Sodium chloride crystal has alternately Sodium and Chloride ions. If an electric field is applied on the crystal, the charges would oscillate back and forth. The natural frequency is in Infrared range.
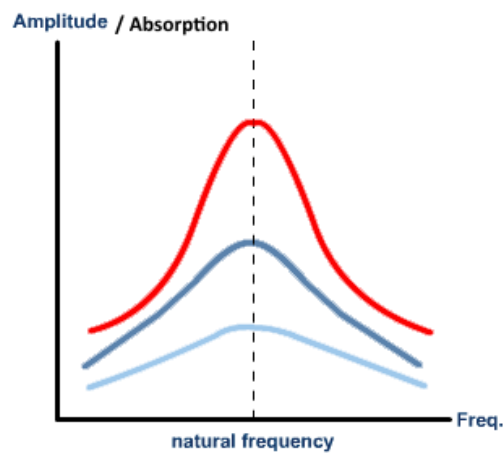
3. LC circuit in a radio tuner is tuned to radio station frequency (say 91.5 MHz radio city) matches with Natural frequency $\left(f = \dfrac{1}{2\pi\sqrt{LC}} \right)$ for amplification of signals.

4. Helmholtz resonator – It is used to analyze complex note. It consists of a hollow sphere of thin glass or brass with an opening through a narrow neck. It is filled with air. The opening receives exciting sound waves and the ears are kept close to the neck. When air is pushed into the sphere and released, the pressure will drive it out. The volume of air in the container behaves as a mass on a spring which is pulled down and released. Compressed air tends to move out and creates low pressure inside. The air will oscillate into and out of the container at its natural frequency given by the expression

$$f = \frac{V}{2\pi}\sqrt{\frac{A}{lv}}$$

V is the velocity of sound, l the length, A the area of the opening, v the volume of the resonator
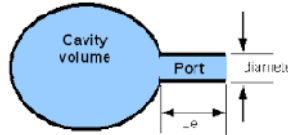
Quality factor

It is customary to describe the amount of damping with a quantity called the quality factor (Q). It is defined as the number of cycles required for the energy to fall off by a factor of 535. (The origin of this numerical factor is $e^{2\pi}$). A mechanical device that can vibrate for many oscillations before it loses a significant fraction of its energy would be considered a high-quality device. Let W be the amount of work done by friction in the first cycle of oscillation, i.e., the amount of energy lost to heat. Let the original energy be E.

$$\frac{1}{Q} = \frac{1}{2\pi}\left(\frac{W}{E}\right)$$

The Full Width at Half Maxima (FWHM) of a resonance is related to its Q and its resonant frequency fres by the equation

$$FWHM = frequency\ Q$$
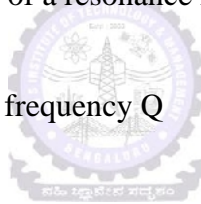
Shock waves

Shock waves: Fluid dynamics is the study of fluids at rest or in motion. It has traditionally been applied in such areas in the design of canal, dam systems, pumps, compressors in refrigerators, air conditioning systems, aerodynamics of automobiles and air planes, ocean phenomena such as tornadoes, hurricanes, tsunami, blood flow, air circulation in our body, lubrication in rotating MP3 disc player etc.

Fluid is a substance that deforms continuously under the application of tangential stress. Liquids and gases are the forms that fluids can take.

Uniform flow: Velocity remains constant.

Steady flow: Velocity at each point in the flow remains constant with time.

Stream lines : Lines drawn parallel to the direction of flow of fluid.

Newtonian fluid: Fluids where shear stress is proportional to deformation.

Laminar flow: Fluid particles move in smooth layers.

Turbulent flow: Fluid particles rapidly mix as they move due to random velocity of fluctuation.

Drag: It is the component of the force on body acting opposite to the direction of motion. Consider a ball flying through the air, in addition to gravity, the ball experiences aerodynamic drag of the air. It is due to pressure build up in the front of the ball as it pushes the air out of the way. Now look at a dust particle falling under gravity at a terminal velocity of 1cm/s, it experiences viscous drag (due to viscosity of air) rather than aerodynamic drag.

Shock: What we perceive as sound generally consists of pressure pulses that move through air. When air undergoes large and rapid compression (following an explosion/the release of engine gases in to an exhaust pipe/when an air craft or a bullet flies at supersonic velocity) a thin wave of large pressure change is produced. This discontinuity is pressure propagates as a wave known as shock wave. A shock wave develops when the flow is supersonic.

Shock tube: A long cylinder is portioned with a cellophane film to give a pressure difference between the two sections. When the partition is ruptured, shock wave develops. The shock wave in this case is at right angles to the flow and is called a normal shock wave.

Mach number: M = Velocity of fluid/velocity of sound.

Subsonic speed	$V_{object} < V_{sound}$	Mach number <1
Sonic speed	$V_{object} = V_{sound}$	Mach number = 1
Supersonic speed	$V_{object} > V_{sound}$	Mach number >1
Transonic		Mach number 0.8 -1.2
Hypersonic		Mach number >5

Reynolds Number:

To compare viscous and aerodynamic drag, Reynold's number is used.

Reynold's number $R_Y = \dfrac{\rho v L}{\mu}$

Here ρ is fluid density, μ is viscosity, v is velocity and L is size scale of the flow.

Large Reynold's number indicates high aerodynamic drag.

Mach number is the ratio of velocity of fluid causing the shock wave generation to the velocity of sound in the medium. It represents the compressibility nature of the medium.

Ultrasonic wave: Sound waves of frequency greater than 20,000Hz.

Acoustic waves: A longitudinal wave that consists of a sequence of pressure pulses propagating in a medium. The speed of an acoustic wave in a material medium is determined by the temperature, pressure, and elastic properties of the medium.

Subsonic waves: These are sound waves with Mach number less than 1. Velocity of the object is less than velocity of sound.

Ex: Low intensity shock waves produced during the motion of ordinary aircrafts.

Supersonic waves: These are shock waves with Mach number greater than 1. Velocity of the object is greater than velocity of sound.

Ex: shock waves produced during the motion of jet planes, bullets etc.

Transonic waves: These are shock waves with Mach number less than 1in the range 0.8 to 1.2

Hypersonic waves: These are shock waves with Mach number greater than 5.

Mach angle: Shock waves propagate as a cone. The semi vertical angle of the cone of shock waves is known as Mach angle (μ). $\mu = \sin^{-1}(1/M)$

Control Volume : It is a volume through which the fluid flows. The boundary of this volume is the physical boundary of the region through which the fluid flows. The equation of continuity, energy flow, variation in pressure and volume are determined with respect to this region.

Important expressions used in fluid dynamics:

Ideal gas equation: $pv = n\,RT$ or $p = \rho RT$

Here p is pressure , v is volume, ρ is density, T is absolute temperature

Equation of continuity/Law of conservation of mass: $\rho VA = $ constant

Here V is velocity of the fluid, A is are of cross section

Law of conservation of energy / Bernoullie's equation :

$$\frac{p}{\rho} + \frac{V^2}{2} + gh = \text{Cons} \tan t$$

$$\frac{\rho V^2}{2} + p + \rho gh = \text{Cons} \tan t$$

$$\frac{k}{k-1} RT + \frac{1}{2} u^2 = \text{cons} \tan t$$

$$\frac{k}{k-1} \frac{p}{\rho} + \frac{u^2}{2} = \text{cons} \tan t$$

k is the ratio of specific heat at constant pressure to specific heat at constant volume

Law of Conservation of Momentum

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

Derivation of Rankine –Hugoniot equation *(Ref: Fluid Mechanics – Nakayama)*

This equation relates pressure, density, temperature ahead and behind a shock wave.

Consider a shock tube with partition separating two regions as shown in the figure.

Continuity equation $\quad \rho_1 u_1 = \rho_2 u_2 \quad$ since area A is constant in this case ………….(1)

Equation for Energy conservation $\quad \dfrac{k}{k-1} \dfrac{p_1}{\rho_1} + \dfrac{u_1^2}{2} = \dfrac{k}{k-1} \dfrac{p_2}{\rho_2} + \dfrac{u_2^2}{2}$ ………………(2)

Equation for momentum conservation $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \qquad$ ………………(3)

From (2)

$$u_1^2 - u_2^2 = \frac{2k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right)$$

From (1) and (3) ....(4)

$$u_1^2 = \frac{p_2 - p_1\rho_2}{\rho_2 - \rho_1\rho_2} - \qquad u_2^2 = \frac{p_2 - p_1\rho_1}{\rho_2 - \rho_1\rho_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{[(k+1)/(k-1)](p_2/p_1)+1}{(k+1/k-1)+p_2/p_1} = \frac{u_1}{u_2}$$

$$\sin ce \ p = \rho RT$$

From (4)

$$\frac{T_2}{T_1} = \frac{(k+1/k-1)+p_2/p_1}{(k+1/k-1)+p_1/p_2}$$

Reddy shock tube:

A shock tube is a device used to study the changes in pressure & temperature which occur due to the propagation of a shock wave. A shock wave may be generated by an explosion caused by the buildup of high pressure which causes diaphragm to burst.

It is a hand driven open ended shock tube. It was conceived with a medical syringe. A plastic sheet placed between the plastic syringe part and the needle part constitutes the diaphragm.
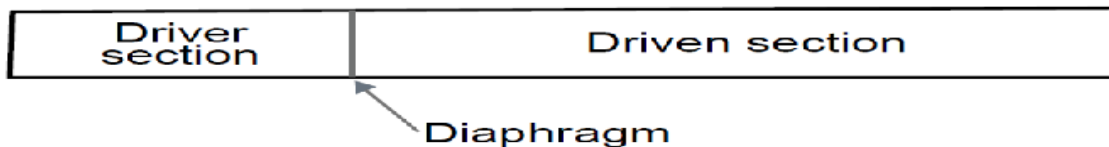
- A high pressure (driver) and a low pressure (driven) side separated by a diaphragm.
- When diaphragm ruptures, a shock wave is formed that propagates along the driven section.
- Shock strength is decided by driver to driven pressure ratio, and type of gases used.
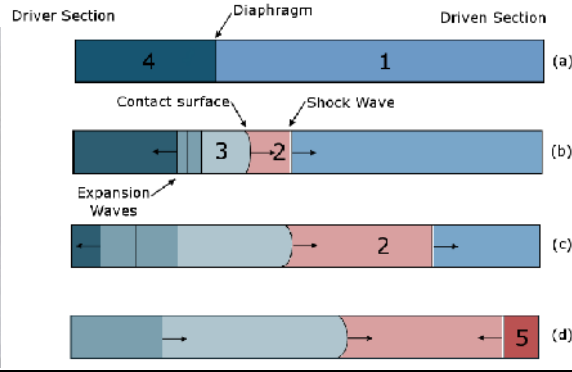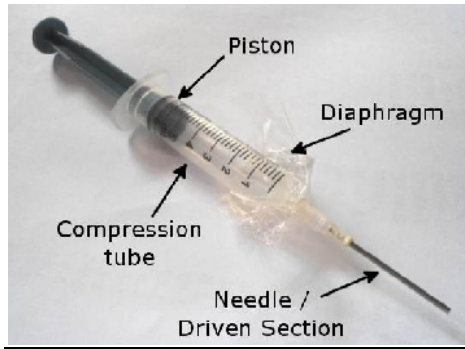
Working:

- The piston is initially at rest and accelerated to final velocity V in a short time t.
- The piston compresses the air in the compression tube. At high pressure, the diaphragm ruptures and the shock wave is set up. For a shock wave to form, $V_{piston} > V_{sound}$.

Formation of shock wave:

As the piston gains speed, compression waves are set up. Such compression waves increase in number. As the piston travels a distance, all the compression waves coalesce and a single shock wave is formed. This wave ruptures the diaphragm.
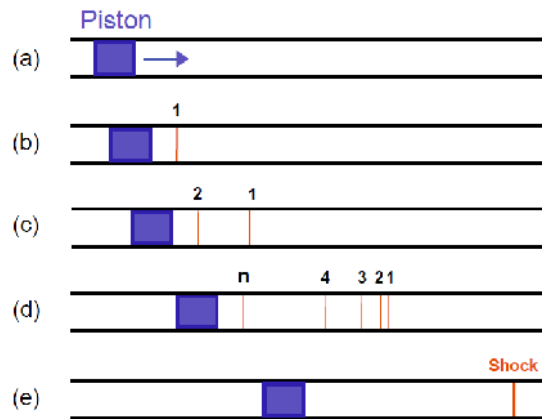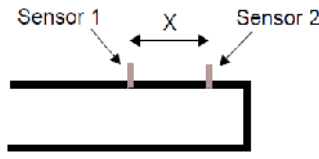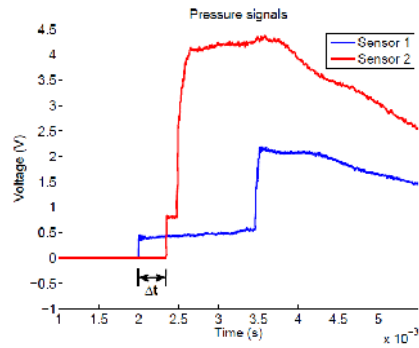
$$U_s = \frac{X}{\Delta t}$$

<u>Mach number</u> = Velocity of the shock wave U_s/Velocity of sound

<u>Uses:</u>

- Aerodynamics – hypersonic shock tunnels, scramjet engines.
- High temperature chemical kinetics – ignition delay
- Rejuvenating depleted bore wells
- Material studies – effect of sudden impact pressure, blast protection materials
- Investigation of traumatic brain injuries
- Needle-less drug delivery
- Wood preservation

Module-2

ELASTIC PROPERTIES OF MATERIALS

One of the most fundamental question that an engineer has to know is how a material behaves under stress, and when does it break. Ultimately, it is the answer to those two questions which would steer the development of new materials, and determine their survival in various environmental and physical conditions.

Elasticity is an elegant and fascinating subject that deals with determination of the stress, strain and displacement distribution in an elastic solid under the influences of external forces. Following the assumptions of linear, small deformation theory, the formulation establishes a mathematical model providing solutions to problems that have applications in many engineering and scientific fields. Civil engineering applications include stress and deflection analysis of structures like rods, beams, plates, shells, soil, rock, concrete, asphalt. Mechanical engineering uses elasticity in numerous problems of thermal stress analysis, fracture mechanics, fatigue, design of machine elements. Materials engineering uses elasticity to determine the stress fields of crystalline solids, dislocations, micro structures. Applications in Aeronautical engineering include stress fluctuations, fracture, fatigue analysis in aerostructures. The subject also provides the basis for study of material behavior in plasticity, viscoelasticity.

Definitions

Stress: Restoring force per unit area

Strain: Ratio of change in dimension to original dimension

 Hooke's Law: For sufficiently small stresses, strain is proportional to stress; the constant of proportionality known as modulus of elasticity depends on the material being deformed and on the nature of the deformation.

Elasticity: The property of materials which undergo deformation under stress and regain their original dimension.

Ex: Spider web, Steel, Graphene

Linear strain (α) - It is the increase per unit length per unit tension along the force

Lateral   strain (β) - It is the lateral contraction per unit length per unit tension perpendicular to force

The **elastic limit** of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length.

Young's Modulus of Elasticity = Longitudinal stress / Linear Strain = $\dfrac{FL}{Al}$

Longitudinal stress or tensile stress is applied along the length and hence causes change in length. Linear strain is the ratio of change in length to original length

Rigidity Modulus of Elasticity = Tangential stress / shear Strain = $\dfrac{F}{A\theta}$

Shearing stress is applied tangential to a surface. As a result, one surface is displaced with respect to another fixed surface. The ratio of displacement to perpendicular distance between the two surfaces is known as shearing strain.

Shearing strain $\quad \theta = \dfrac{l}{L} \quad$ when ө is small

Bulk Modulus of elasticity = Normal stress / Volume Strain = $\dfrac{FV}{Av}$

Application of normal (compressive) stress causes change in volume. Volume strain is the ratio of change in volume to original volume.

G

Plasticity: The property of materials which undergo deformation under stress and do not regain their original dimension.

θ

HOOKE's LAW

A – Elastic Limit

B – Yielding Point

C - D – Plastic range- Strain hardening

Stress is proportional to strain at smaller magnitudes. As the stress is increased to large magnitudes strain increases more rapidly and the linear relationship between stress and strain ceases to hold. This is referred as elastic limit (A). After the yielding point (B), the strain randomly increases. This may lead to strain softening in some materials. C on wards the material attains permanent status (Plastic) and is known as strain hardening region. After D, the material breaks.

Effect of stress – Temperature – Annealing – Impurities on Elasticity

Effect of stress:
1) Elastic fatigue
Elastic properties of a body repeatedly subjected to stress show random variation.
Ex: Piston and connecting rods in a locomotive are subjected to repeated tensions and compressions during each cycle. Their elastic properties randomly fluctuate. It may break under a stress less than elastic limit.
2. Annealing : Annealing operation involves heating and gradual cooling. The crystal grains form a uniform orientation forming larger domains. This causes decrease in elastic

properties. Operations like hammering, rolling break up the crystal grains resulting in increase of elastic properties.

3. Temperature : Inter molecular forces decreases with rise in temperature. Hence the elasticity decreases with rise in temperature. (But the elasticity of invar steel (alloy) does not change with change of temperature).Carbon filament which is highly elastic at ordinary temperature, becomes plastic when heated.
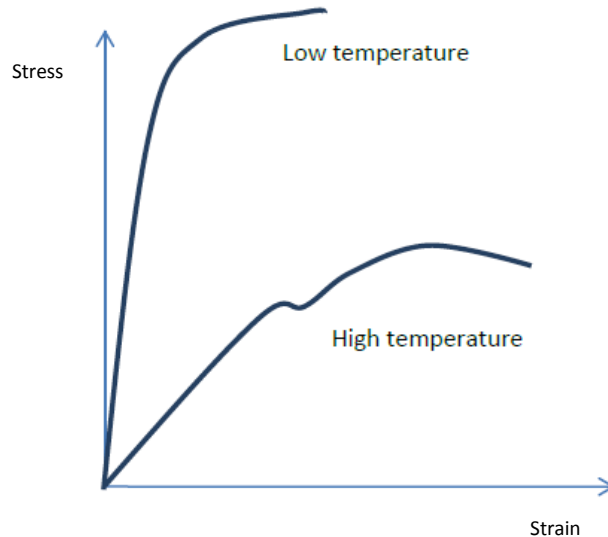
4. Impurities : Presence of impurities alters elasticity. It can increase or decrease depending on the nature of impurities. Carbon is added in minute quantities to molten Iron to increase its elastic property.

FACTOR OF SAFETY
To avoid permanent deformation due to maximum stress, the engineering tools are to be used within the elastic limit with a working stress.

Factor of safety = Breaking stress / Working stres

RELATION BETWEEN BULK MODULUS (K) - α –β
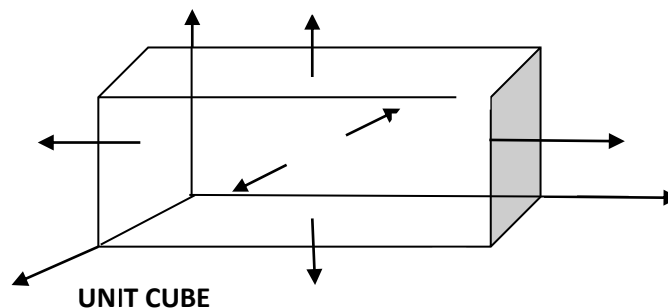
UNIT CUBE

Let stresses T_X, T_Y and T_Z act perpendicular to faces of a unit cube as shown in the figure .Let α be the increase per unit length per unit tension (linear strain) along the force, β be the lateral contraction (lateral strain) per unit length per unit tension perpendicular to force.

Elongation produced along X axis = $T_X \cdot \alpha \cdot 1$

Contraction produced along X axis $= (T_y \cdot \beta \cdot 1 + T_z \cdot \beta \cdot 1)$

Length of AB $= 1 + T_x \alpha - T_y \beta - T_z \beta$

Length of BE $= 1 + T_y \alpha - T_x \beta - T_z \beta$

Length of AB $= 1 + T_z \alpha - T_y \beta - T_x \beta$

Volume of cube $= (1 + T_x \alpha - T_y \beta - T_z \beta) \ X \ (1 + T_y \alpha - T_x \beta - T_z \beta) X (1 + T_z \alpha - T_y \beta - T_x \beta)$

$= 1 + T_X \alpha + T_Y \alpha - T_Z \beta - T_X \beta - T_y \beta + T_Z \alpha - T_x \beta - T_Y \beta - T_Z \beta$

$= 1 + (\alpha - 2\beta)(T_X + T_Y + T_Z)$ neglecting terms containing $\alpha.\beta, \alpha^2, \beta^2$

$= 1 + (\alpha - 2\beta)(3T)$ if $T_X = T_Y = T_Z$

Increase in volume = $= 1 + (\alpha - 2\beta)(3T)$ - 1

If Inward pressure is applied, **reduction in volume** $= 1 + (\alpha - 2\beta)(3P)$

$$\text{Bulk Modulus} = \frac{P}{3P(\alpha - 2\beta)} = \frac{1}{3(\alpha - 2\beta)}$$

RIGIDITY MODULUS
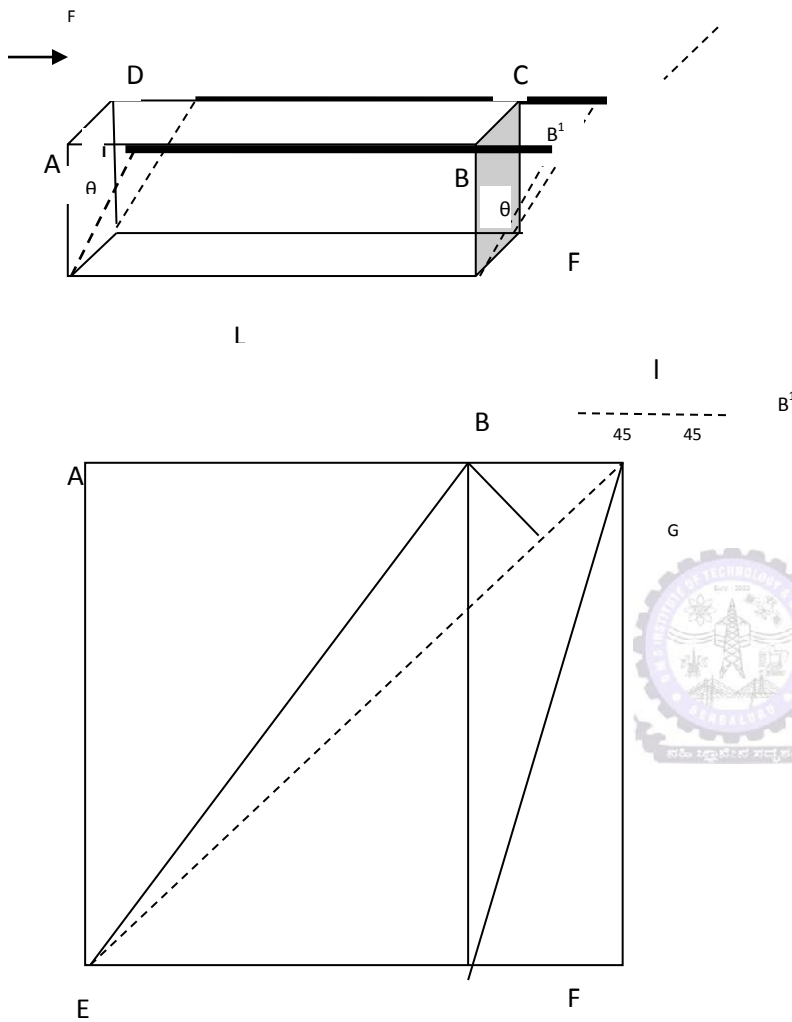
Let the face ABCD of a cube of side L be sheared by a Force F through an angle θ.

Shearing stress $= \dfrac{F}{L^2} = T$

Shearing Strain $= \dfrac{l}{L} = \theta$

Rigidity Modulus $= \dfrac{T}{\theta}$

Shearing stress along AB is equivalent to expansive stress along EB and compressive stress along AF. Let α be the longitudinal expansive strain per unit Stress per unit length and β be the lateral compressive strain per unit stress per unit length respectively.

Elongation along EB$= EB.\alpha.T.$

Compression along AF $= AF..T.\beta$

Net extension EB^1= $L.\sqrt{2}.T(\alpha + \beta))$

Also , from right angled triangle BB^{1}G,

Elongation EB^1= $\dfrac{l}{\sqrt{2}}$

$$L.\sqrt{2}.T(\alpha + \beta)) = \dfrac{l}{\sqrt{2}}$$

$$\dfrac{T}{\dfrac{l}{L}} = \dfrac{1}{2(\alpha + \beta)}$$

$$n = \dfrac{1}{2(\alpha + \beta)}$$

RELATION BETWEEN ELASTIC CONSTANTS

$$\alpha - 2\beta = \frac{1}{3K} \quad ......(1)$$

$$\alpha + \beta = \frac{1}{2n} \quad ....(2)$$

$$(2) - (1)$$

$$3\beta = \frac{1}{2n} - \frac{1}{3K}$$

$$\beta = \frac{3K - 2n}{18nK}$$

$$2.(2) + (1)$$

$$\alpha = \frac{3K + n}{9Kn}$$

$$Y = \frac{1}{\alpha} = \frac{\sigma}{\beta}$$

$$\frac{9}{Y} = \frac{3}{n} + \frac{1}{K}$$

POISSON RATIO

When a material is stretched, the increase in its length (α) is accompanied by decrease in cross section (lateral strain β). Within the elastic limit, the lateral strain is proportional to longitudinal strain and the ratio between them is a constant for a material known as Poisson ratio.

$$\sigma = \frac{\beta}{\alpha}$$
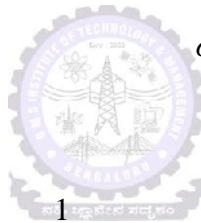
RELATION BETWEEN K - n - σ

$$K = \frac{1}{3(\alpha - 2\beta)}$$

$$K = \frac{1}{3\alpha(1 - 2\sigma)}$$

$$K = \frac{Y}{3(1 - 2\sigma)}$$

$$n = \frac{1}{2(\alpha + \beta)}$$

$$n = \frac{1}{2\alpha(1 + \sigma)}$$

$$n = \frac{Y}{2(1 + \sigma)}$$

$$\sigma = \frac{3K - 2n}{6K + 2n}$$

LIMITS OF σ

$$3(1 - 2\sigma) = 2n(1 + \sigma)$$

1. If σ be a positive quantity, (1-2σ) should be positive

    2σ<1

    σ < 0.5

    When σ =0.5, the material is said to be incompressible

2. If σ be a negative quantity, $(1 + σ)$ should be positive

$$σ < -1$$

Resilience: Capacity to resist a heavy stress without acquiring permanent elongation.

COUPLE PER UNIT TWIST OF A SOLID CYCLINDER
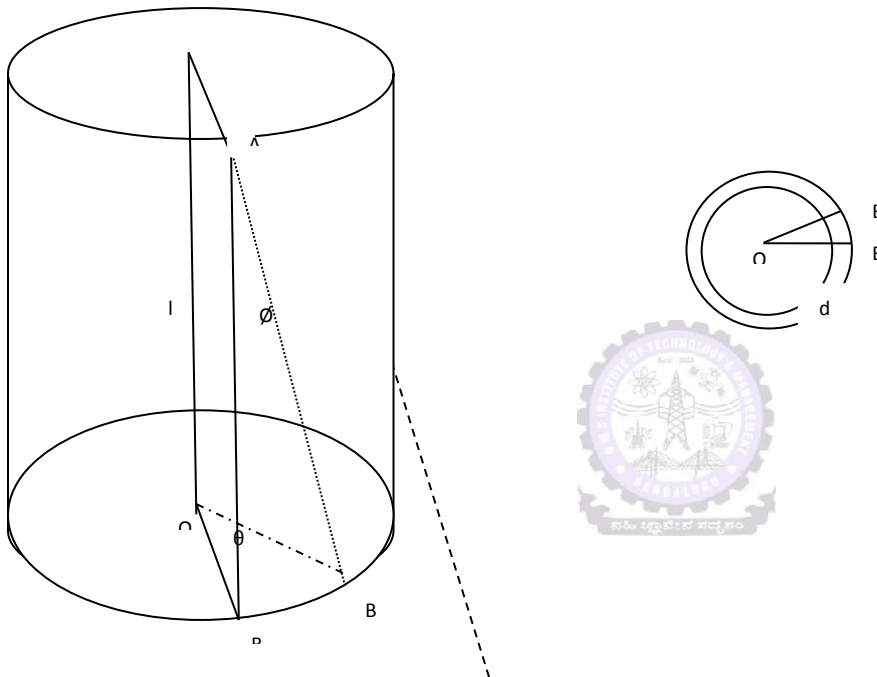
Consider a cylindrical rod of rigidity modulus n, length l, radius r fixed at one end and twisted at the other end through an angle θ by a couple. Imagine the cylinder to be made of large number of coaxial cylinders of increasing radius. Consider a cylinder of radius x and thickness dx. For a given couple, the displacement at its rim is maximum. On twisting, the point B shifts to B^1.

$$BB^1 = l\phi = x\theta$$

$$\phi = \frac{x\theta}{l}$$

$$n = \frac{F}{\phi} = \frac{nx\theta}{l}$$

This force is acting on the area 2.πx.dx

Total force $\qquad F = \dfrac{nx\theta}{l} 2\pi x . dx$

Moment of force along OO^1 $\qquad = couple = \dfrac{nx^3\theta}{l} 2\pi . dx$

Total twisting couple $= \displaystyle\int_0^R \dfrac{nx^3\theta}{l} 2\pi . dx = \dfrac{2\pi n\theta}{l}\dfrac{R^4}{4}$

Couple per unit twist $== \dfrac{\pi n}{l}\dfrac{R^4}{2}$

TORSIONAL PENDULUM

A heavy object suspended from end of a fine wire rotating about an axis constitutes a torsional pendulum. Let θ be the twisting angle .The restoring couple set up in it is equal to $\dfrac{\pi n\theta}{l}\dfrac{R^4}{2}$ .This produces angular acceleration.

If I is the moment of Inertia of the object

$$\text{Torque} = I . \dfrac{d\omega}{dt} = -\dfrac{\pi n\theta}{2l}R^4$$

$\dfrac{d\omega}{dt} = -\dfrac{\pi n r^4\theta}{2lI} = -\dfrac{C}{I}\theta \qquad$ [Angular Acceleration α - angular

displacement]

C is the couple per unit twist.

The motion is simple harmonic as shown by the expression torque =

$$\tau = I\dfrac{d^2\theta}{dt^2} = -C\theta$$

$$F = \dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x$$

$$\omega = \sqrt{\dfrac{k}{m}}$$

Similarly, for angular motion,

$$\omega = \sqrt{\frac{C}{I}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{C}{I}}$$

Time period $\quad T = 2\pi\sqrt{\dfrac{I}{C}}$

BENDING OF BEAM

Beam is a bar of uniform cross section whose length is very much larger than thickness. When such a beam is fixed at one end and loaded at the other, the beam is bent under the action of couple produced by the load. Upper surface of the beam gets stretched and lower surface gets compressed. The extension is maximum in the upper most filaments and compression, maximum in the lowermost ones. The surface which does not get affected is known as neutral surface.

If the bending is uniform, the longitudinal filaments get bent into circular arcs in planes parallel to the plane of symmetry (plane of bending).The line of intersection of plane of bending with neutral surface is called neutral axis.
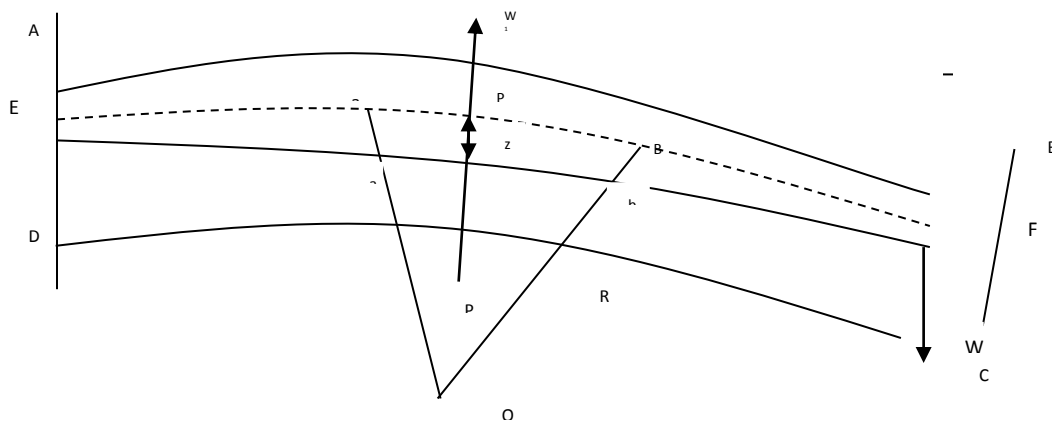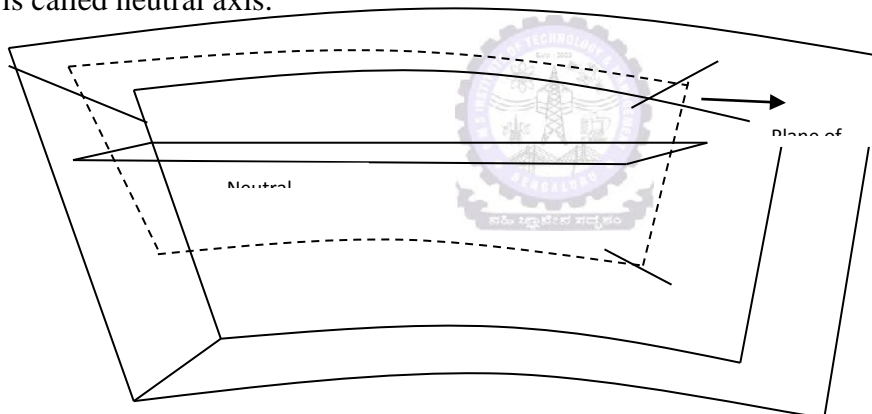
BENDING MOMENT

In the above figure, ABCD is a beam fixed at AD and loaded at B. EF is neutral axis. Whereas the load tends to bend the beam, an equal and opposite reactional force W^1 will be acting upwards along pp^1. These two forces constitute a couple and the moment of this couple is called bending moment.

Let the beam be bent in the form of circular arc subtending angle θ at the centre of curvature O. Let $a^1 b^1$ be an element at a distance Z from the neutral axis.

$$a^1 b^1 = (R + Z)\theta$$

Increase in length of the filament $= a^1 b^1 - ab = (R + Z)\theta - R\theta$

$$\text{Strain} = \frac{Z\theta}{R\theta} = \frac{Z}{R}$$
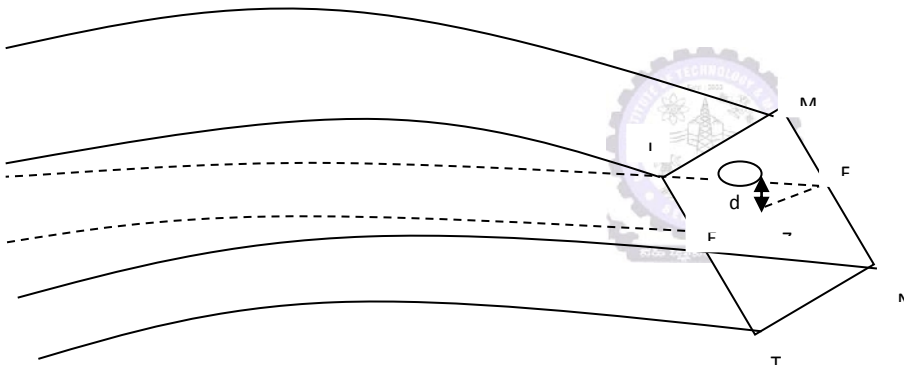
Let LMNT be the rectangular cross section perpendicular to length. EF is the neutral surface. The restoring force on upper half acts inwards and outwards on the lower half.

Consider a small area da at a distance z from the Neutral surface.

$$\text{Strain produced in the filament} = \frac{Z}{R}$$

$$\text{Force on area da} = Y.da.\frac{Z}{R}$$

Moment of this force about the neutral surface $= F.Z = Y.da.\dfrac{Z^2}{R}$

Total moment of forces in LMNT $M = Y.\dfrac{\sum Z^2.da}{R} = \dfrac{Y}{R}I$

Here I is Geometrical Moment of Inertia.

For rectangular cross section

$$\text{area} = \text{b X d}, \quad k = \dfrac{d^2}{12}$$

$$I = ak^2 = \dfrac{bd^3}{12}$$

$$M = \dfrac{Ybd^3}{12R}$$

For Circular cross section

$$\text{area} = \pi R^2, \quad k = \dfrac{r^2}{4}$$

$$I = ak^2 = .\dfrac{\pi R^4}{4}$$

$$M = \dfrac{Y\pi r^4}{4R}$$

CANTILEVER

It is a beam fixed horizontally at one end and loaded at the other.
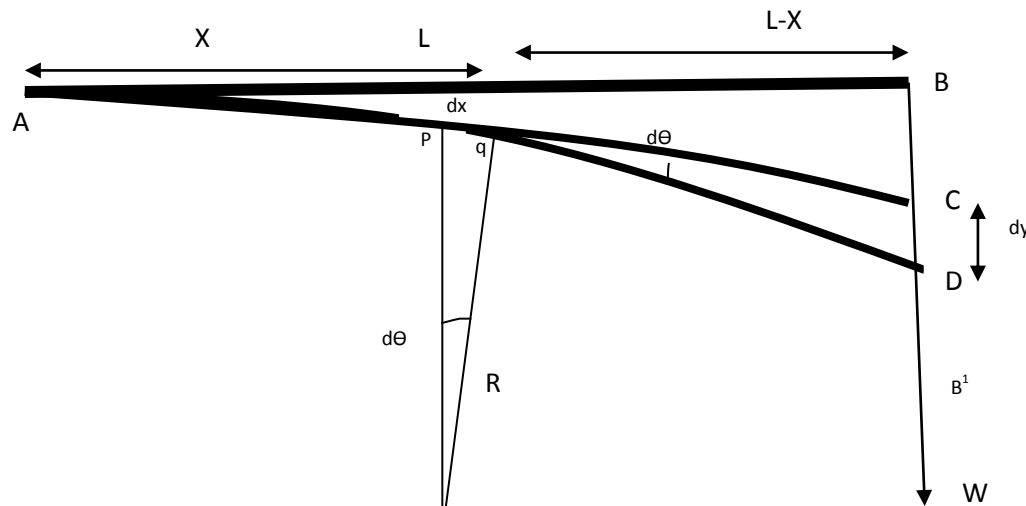
Let AB be the neutral axis of the cantilever of length L fixed at A and loaded at B. Consider a section P of the beam at a distance x from A.

Bending moment = W .PC = W (L-X) $= = Y\dfrac{I}{R} = Ya\dfrac{k^2}{R}$

Here R is the radius of curvature of neutral axis at P. The moment of the load increases towards the point A, the radius of curvature is different at different points and decreases towards A. For a point Q at a distance dx from P, it is same as at P.

$\quad$ PQ = dx = R.dΘ

$$\text{Bending moment} \qquad W (L-X) = Y\frac{ak^2 d\theta}{dx}$$

Draw tangents to the neutral axis at P and Q meeting the vertical line at C and D. The angle subtended by them is dΘ. The depression of Q below P is given by

$$dy = (L-X)d\theta = W\frac{(L-X)^2}{Yak^2}.dx$$

Total depression BB^1 of the loaded end
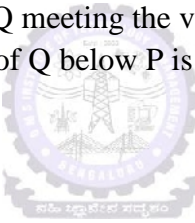
$$\int dy = \int_0^L W\frac{(L-X)^2}{Yak^2}.dx = W\frac{L^3}{3YI}$$

Relation between Shear strain – Longitudinal strain – Compression stain

COROLLARY 1: A shear of Θ is equivalent to an elongation strain $\Theta/2$ and compression strain $\Theta/2$ at right angles to each other.
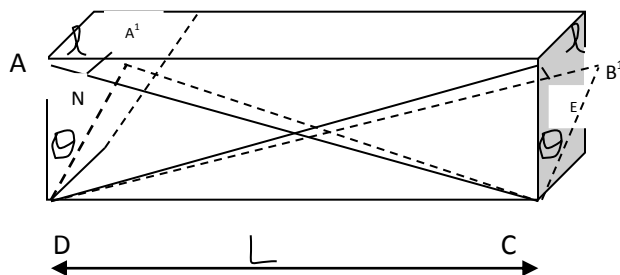
From the triangle BEB^1, $B\hat{B}^1E = 45^0$

$$\cos 45 = \frac{EB^1}{BB^1} \Rightarrow EB^1 = \frac{l}{\sqrt{2}}$$
$$DE = \sqrt{2}L$$

Extension strain along DE $= \frac{EB^1}{DE} = \frac{l}{2L} = \frac{\theta}{2}$ where is the shearing strain

From the triangle AA^{1}N, $AA^1N = 45^0$

$$\cos 45 = \frac{AN}{AA^1} \Rightarrow AN = \frac{l}{\sqrt{2}}$$
$$CN = \sqrt{2}L$$

Compression strain along AC $= \frac{AN}{CN} = \frac{l}{2L} = \frac{\theta}{2}$

Elongation strain + Compression strain = Θ/2 + Θ/2 = Θ

COROLLARY 2

A shearing stress of F is equivalent to an tensile stress F/2 and compression stress F /2 at right angles to each other.[A tangential force of F is equivalent to a tensile force $\frac{F}{\sqrt{2}}$ and compression force $\frac{F}{\sqrt{2}}$ at right angles to each other]
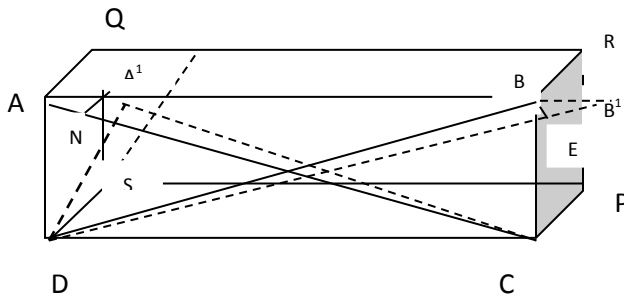
Tensile stress along DB perpendicular to the surface AQPC $= \dfrac{F\cos 45}{L.L\sqrt{2}} = \dfrac{F/\sqrt{2}}{L.L\sqrt{2}}$

Compression stress along AC perpendicular to the surface DBRS $= \dfrac{F\cos 45}{L.L\sqrt{2}} = \dfrac{F/\sqrt{2}}{L.L\sqrt{2}}$

Stain softening

Strain softening is defined as the region in which the stress in the material is decreasing with an increase in strain. This observed in certain materials after yielding point as represented in the diagram. It causes deterioration of material strength with increasing strain, which is a phenomenon typically observed in damaged quasi brittle materials, including fiber reinforced composites and concrete. It is primarily a consequence of brittleness and heterogeneity of the material.
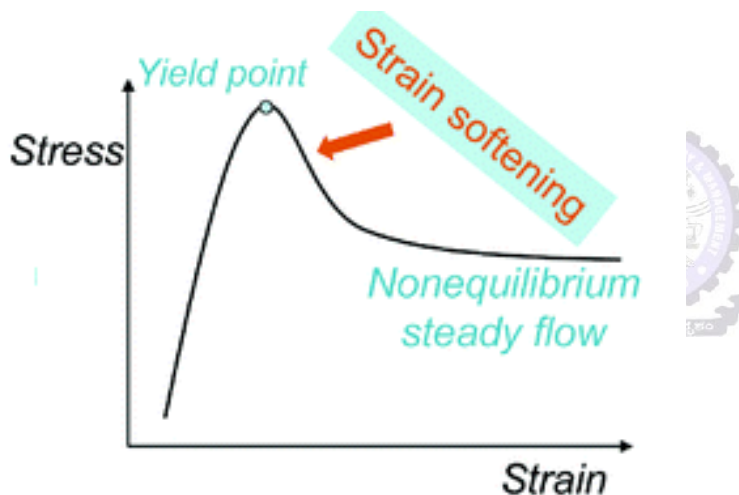
Strain Hardening

When a material is strained beyond the yield point, more and more stress is required to produce additional plastic deformation and the material becomes stronger and more difficult to deform. This is

known   as Strain   Hardening**.**   The

material is permanently   deformed   increasing   its   resistance to further deformation. Strain hardening reduces ductility and increases brittleness.

A material that does not show any strain hardening is said to be perfectly plastic.

The strain hardening exponent (coefficient) is given by the expression $\sigma = K\,\varepsilon^{n}$.

σ is the applied stress

ε is strain

n is the strain hardening coefficient

K is the strength coefficient (elasticity) . It is a measure of the ability of a metal to strain harden.

The value of n lies between 0.1 and 0.5 for most metals.

A material with a higher value of n has a greater elasticity than a material with a low value of n.

Applications of beams

Beams are an integral part of Civil engineering structural elements (bridges, dams, multistoreid buildings), measuring devices (Tunneling microscopes), automobile frames, aircraft components, machine frames. They are designed to withstand heavy load. Cantilever beam, simple support beam (connecting beam to beam), roller beam are generally observed in heavy structures.

Applications of Torsional Pendulum:

1. The working of Torsion pendulum clocks is based on torsional oscillation.

2. The freely decaying oscillation of Torsion pendulum in medium (like polymers),helps to determine their characteristic properties.

3. Determination of frictional forces between solid surfaces and flowing liquid environments using forced torsion pendulums.

4. Torsion springs are used in torsion pendulum clocks.

> **Clothes Pins**. The working of clothes pins is facilitated by the torsion springs. These springs provide an excellent clamping action.

> **Automotive:** Torsion springs are known for providing even tension, along with smooth and frictionless motion. These springs are widely used in the automotive industry for various parts such as a vehicle suspension system, chassis, automotive valves, clutches, and gear shifters.

> **Medical Equipment:** In the medical industry, the torsion springs are used in medical immobilization devices, hospital beds, several dental applications, wheelchair lifts and many more.

> **Door Hinges:** These springs are widely used in different types of door hinges. These springs allow the door to come back to its original position.

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DEPARTMENT OF PHYSICS
COURSE MATERIAL   **SUBJECT: - Engineering Physics**
MODULE -3   **SUBJECT CODE: - 18 PHY 12 /22**
(**Maxwell's equations, EM Waves and Optical fibers**)

Study material

Engineering Physics 2018-19 CBCS Scheme   Name of the Faculty:- Dr. Daruka Prasad B

Module-3

MAXWELL's EQUATIONS – EM WAVES AND OPTICAL FIBERS

INTRODUCTION

Today's cutting edge technology - 'ELECTROMAGNETIC COMMUNICATION' or 'WIRELESS COMMUNICATION' originated during 1865 with the coining of the term ELECTROMAGNETIC WAVES by James Clark Maxwell who is regarded as the founder of electrodynamics.

Maxwell quantified the existing laws of electromagnetism (Gauss law, Amperes law, Faradays law) and also theoretically modified Amperes law for varying current situations. According to his modified Amperes law $\nabla X \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$, varying electric field induces changing magnetic field .That is, magnetic field is produced by conventional flow of current and also by the rate of change of electric displacement .This can be observed in the case of capacitor connected in an AC circuit .So this source of magnetic field $(\dfrac{\partial \vec{D}}{\partial t})$ behaving similar to electric current was given the name 'DISPLACEMENT CURRENT DENSITY'. It was this concept of displacement current density which led to the concept of electromagnetic waves .Hertz made use of this concept to develop his antenna to radiate and receive electromagnetic signals. This culminated in the invention of RADIO by Marconi.

Today as most of us are aware, the transmission of information through electromagnetic waves is the most efficient and useful method of communication.

Now to understand this whole process and to take this further, one must understand the basic laws of electricity like Coulombs law, Gauss law, Laplace Equation which are the techniques of determining electric field at a point due to static charges and Biot- Savartz law, Ampers law which are the methods

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of determining magnetic field due to steady currents. Later, through Maxwell equations we can acquire

an understanding about Electromagnetic waves.

REVIEW OF VECTOR CALCULUS

Cartesian coordinate system:
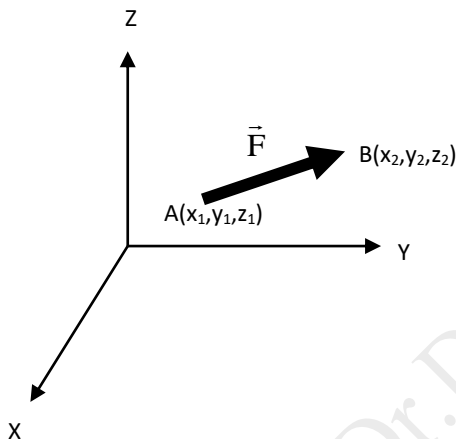
$$\vec{A} = \vec{A}_x \hat{a}_x + \vec{A}_y \hat{a}_y + \vec{A}_z \hat{a}_z$$

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Unit vector along $\vec{A} = \dfrac{\vec{A}}{\left|\vec{A}\right|}$

$$\vec{A}.\vec{B} = |A|.|B|\cos\theta$$

$$\vec{C}X\vec{D} = \left|\vec{C}\right|.\left|\vec{D}\right|\sin\theta.\hat{n}$$

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}A = \frac{\partial A}{\partial x}\hat{a}_x + \frac{\partial A}{\partial y}\hat{a}_y + \frac{\partial A}{\partial z}\hat{a}_z$$

$$\nabla \times A = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

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Del operator:
$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

Divergence: It represents the magnitude of a physical quantity emerging or converging at a point. For example tip of a fountain head is a source of divergence. Electric fields are said to be divergent in nature. Mathematically it is obtained by differentiating components of a vector function $F(F_x, F_y, F_z)$ with respect to position coordinates x,y,z respectively.

$$\nabla \bullet \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Ex: Volume charge density enclosed in a closed surface is expressed as

$$\nabla \bullet \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Right hand side in the above expression is a scalar. Divergence operation on vector yields a scalar function. Divergence of vector is zero if there is no outflow or inflow. Magnetic fields form closed loops and their divergence is zero. $\nabla \bullet B = 0$

Diverging electric field lines from positive charge is an example for Positive divergence

converging electric field lines from negative charge is an example of negative divergence

Gradient : This operation is performed to study spatial variation of a scalar variable. It yields a vector function**.**

Ex: Variation of temperature in a room is represented as

$$\nabla T = \frac{\partial T}{\partial x}\hat{a}_x + \frac{\partial T}{\partial y}\hat{a}_y + \frac{\partial T}{\partial z}\hat{a}_z$$

The right hand side of this expression is a vector. It is in the direction of maximum increase / decrease of the scalar variable.

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Electric field is represented as gradient of scalar vector potential

$$E = -\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

Curl : This operation degree of rotation per unit area. It yields a vector.

$$\nabla \times A = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Ex: Magnetic field around a straight conductor carrying current is expressed as
$$\nabla X H = J$$

Curl of a irrotational vector is zero. Static electric fields posses no curl. $\nabla X E = 0$

Vector joining two points:
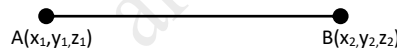
$$\vec{AB} = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

$$\left|\vec{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Vector identities:

$$\nabla \times (\nabla \times A) = \nabla.(\nabla.A) - \nabla^2 A$$
$$A.(B \times C) = B.(C \times A) - C(A \times B)$$
$$A \times (B \times C) = B.(A.\ C) - C(A.\ B)$$

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Dot Product:

Dot product of two vectors gives a scalar.

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$$

Dot product of A and B represents component of A in the direction of B.

Generally used to find the component of a force in a given direction – a necessity in electrostatics. It is a measure of parallelness of two vectors

Ex:

Work done $= \vec{F} \bullet \hat{x}$ (Force X displacement)

Potential $= \vec{E} \bullet \vec{dl}$ (Electric field X distance)

Rate of flow of water $= V \bullet A$ (Velocity X Area)

Cross Product:

Cross product of two vectors gives a scalar.

$$\vec{A}X\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta.\hat{n}$$

Ex: Force on a current carrying conductor kept in magnetic field $= \vec{F} = (\vec{B}XI)L$

For an electromagnetic wave,   Poynting vector P $= \vec{P} = \vec{E}X\vec{H}$

For a particle moving along a circle,   Linear velocity $\vec{V} = \vec{r}X\vec{\omega}$

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Generally used to find a vector $\perp$ to a plane- a relevant feature of magnetostatics.

Line integration: Integration in one dimension

Ex: If the charge density along a line charge is ρ_l, then the total charge on a line is given by

$$q = \int_0^L \rho_l \, dl$$

Workdone to move a unit charge in an electric field is given by $W = \int_A^B E \bullet dl$

Surface Integration: Integration in two dimension

Ex: If the charge density on a uniform surface is ρ_s, then the total charge on a surface is given by

$$Q = \iint \rho_s \, dx.dy$$

Volume Integration: Integration in three dimension

Ex: If the charge density in a volume is ρ_v, then the total charge in a volume is given by

$$Q = \iiint_v \rho_v \, dx.dy.dz$$

Review of Coulomb's law:
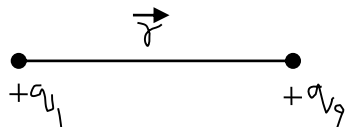
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The electrostatic force between two charged bodies is

a. directly proportional to the product of their charges

b. inversely proportional to the square of the separating distance

c. acts along the position vector joining the two charges

d. is attractive for unlike charges and repulsive for like charges.

Force on charge q_2 due to q_1 separated by a distance r is

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r} \quad \text{newton}$$

where ε is the permittivity of the medium, $\varepsilon = \varepsilon_o \cdot \varepsilon_r$ $\varepsilon_{o} = 8.85\times10^{-12}$ F/m

ε_r is the relative permittivity

q_1 and q_2 are in coulombs

r is in metres

Vectorial form:

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$
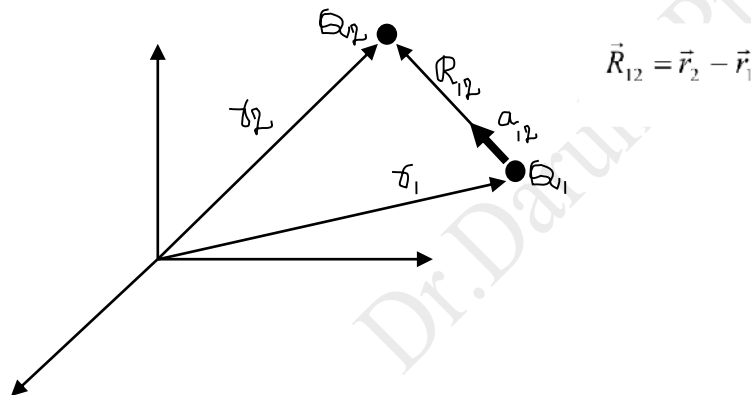
Coulomb's law is given by $\vec{F} = \dfrac{1}{4\pi\varepsilon} \dfrac{q_1 q_2}{|\vec{R}_{12}|^2} \hat{a}_{12}$

Where $\hat{a}_{12}$ is the unit vector in the direction of R_{12} $= \dfrac{\vec{R}_{12}}{|\vec{R}_{12}|}$

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Study material

Electrical field Intensity:

It is the force experienced by a unit positive charge.

$$\vec{E} = \underset{q \to 0}{Lt} \frac{F}{q}$$

In the above diagram, the electric field intensity at q_2 due to q_1 is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r} \left(\frac{1}{q_2} \right) = \frac{q_1}{4\pi\varepsilon |\vec{r}|^2} \hat{r} \quad \frac{N}{C} \quad \text{or} \quad \frac{V}{m}$$

Electric field is a vector. It can be represented as the gradient of potential along the direction of decreasing potential.

$$\vec{E} = -\frac{dV}{dx} = -\nabla V \qquad V/m$$

In Cartesian coordinate system

$$\vec{E} = -[\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial v}{\partial z} \hat{a}_z]$$

GAUSS LAW

It is a technique to evaluate electric intensity at a point due to a **symmetric charge distribution**. Gauss quantified Faradays postulates on electric flux.

Electric flux (Φ): Lines of force normal to the surface.

Lines of force emanate from positive charge and converge on negative charge.

Faradays Law of electric flux (not to be confused with the law of electromagnetic induction):

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The number of lines of force emanating from a charge is directly proportional to the quantity of charge.

Electric flux density (D):[Electric displacement]

It represents the lines of force crossing unit area normally.

$$\vec{D} = \frac{flux(\Phi)}{area(A)}$$

If the flux lines are inclined to the normal, then $\vec{D} = \frac{flux(\Phi)}{area(A) \bullet \cos\theta}$

$$\text{Also} \quad \vec{D} = \varepsilon \vec{E}$$

Statement of Gauss law:

The total electric flux over a closed surface $\left(\oiint_s \vec{D} \bullet \vec{ds} \right)$ is equal to the charge enclosed by the surface.

$$\text{Total electric flux } \Phi = \left(\oiint_s \vec{D} \bullet \vec{ds} \right) = q$$

Proof:
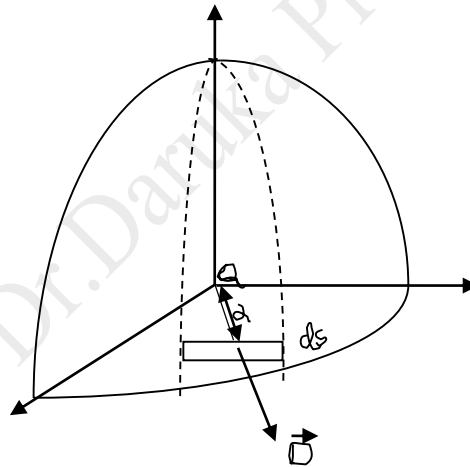
Imagine a point charge 'Q' kept at the origin of a sphere of radius 'r'. Let the flux density passing

normally through the surface be $\vec{D}$.

Total flux leaving the entire spherical surface is

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$$\Phi = \left(\oiint_s \vec{D} \bullet \vec{ds} \right)$$

From the definition of flux density,

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_n$$

$$\therefore \phi = \frac{Q}{4\pi r^2} \iint_s ds \hat{a}_n = \frac{Q}{4\pi \, r^2} * 4\pi \, r^2$$

Total flux $\Phi = Q$

Differential or point form of gauss Law:

$$\nabla \bullet \vec{D} = \rho_v$$

Gauss divergence theorem:

Statement: The volume integral of the divergence of a vector function 'F' over a volume 'V' is equal to the surface integral of the normal component of the vector function 'F' over the surface enclosing the volume V.

Explanation:

Consider a Gaussian surface enclosing a charge Q with a charge density ρ_v.

$$\text{Then } Q = \int_v \rho dv$$

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From Gauss law, total charge enclosed $= \therefore Q = \iint \vec{D} \bullet ds$

From differential form of Gauss law, $\nabla \bullet \vec{D} = \rho_v$

$$\therefore Q = \int_v \nabla \bullet \vec{D}\, dv = \int_s \vec{D} \bullet ds$$

$$\boxed{\therefore \int_v \nabla \bullet \vec{D}\, dv = \int_s \vec{D} \bullet ds}$$

Stokes theorem: Curl of a vector over a surface is equal to the line integral of the vector along the line enclosing the surface

$$\iint_s \nabla XH.ds = \int_l H.dl$$

Biot – Savartz law:

This law gives the magnetic flux density at a point due to a current carrying conductor.

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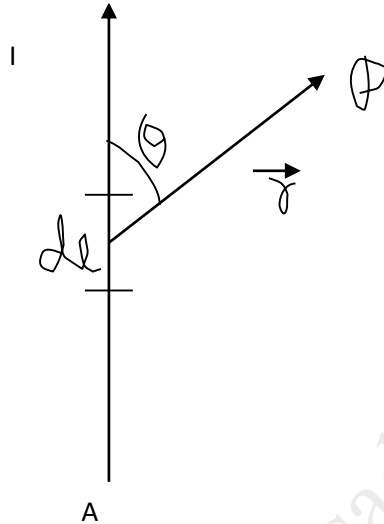
Let AB be a conductor carrying a current I .The magnetic flux density at a point P due to a current element dl is obtained from Biot-Savartz law as

$$\vec{dB} = \frac{\mu}{4\pi}\frac{I\,\vec{dl}\,X\,\hat{a}_r}{r^2}$$

The direction is such that the flux density is perpendicular to the plane containing the current element and the position vector to the point at which the flux density is calculated.

Statement: The magnitude of the flux density at a point due to a current element is directly proportional to the product of the current (I) , the length of the current element(dl) and the sine of the angle(sinθ) between the current element and the position vector to the point from the current element and it is inversely proportional to the square of the distance between the point and the current element.

Gauss law for magnetic fields:

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Magnetic flux lines always form a closed loop. In the case of a closed surface in a magnetic field, every line that enters the surface, there must always be a line emerging out of the surface.

Therefore total flux over the closed surface = 0.

$$\phi = \int \vec{B} \bullet \vec{ds} = 0$$

$$\Rightarrow \nabla.\vec{B} = 0$$

Divergence of magnetic field is zero. [There is no source or sink for magnetic field].They form closed loops. Hence magnetic fields are said to be solenoidal.

Ampere's law:

Amperes law states that the line integral of magnetic flux density ($\vec{B}$) over any closed path is equal to the product of current enclosed by the path and the permeability of the medium(μ).

$$\oint \vec{B}.\vec{dl} = \mu I$$

or $$\oint \vec{H}.\vec{dl} = I \qquad\qquad \because \vec{B} = \mu \vec{H}$$

Faradays law of electromagnetic induction:

According to this law, the magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through it. The induced emf will be in a direction which opposes the change which causes it.

Induced emf e = - $\dfrac{d\varphi}{dt}$ where Φ is the flux linking with the circuit.

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Note: Please refer APPENDIX for class room demonstration of Faradays law

Integral form of Faradays Law:

Consider a closed loop subjected to change in magnetic flux Φ. The emf induced is given by

$$\text{e} = -\frac{d\varphi}{dt} = \oint E.dl$$

$$\oint E.dl = \text{e} = -\frac{d\varphi}{dt} = -\int \frac{d\vec{B}}{dt}.\vec{ds} \quad INTEGRAL\ FORM.$$

Differential form of Faradays Law:

From Stokes theorem $\quad \oint E.dl = \int_s \left(\nabla x \vec{E} \right).ds$

The above equation becomes

$$\int_s \left(\nabla x \vec{E} \right).ds = -\int \frac{d\vec{B}}{dt}.\vec{ds}$$

$$\therefore \nabla x \vec{E} = -\frac{d\vec{B}}{dt}$$

MAXWELL EQUATIONS - Differential Form

The following are the four Maxwell's equations for **static fields**.

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$$\textbf{1.} \nabla \bullet \vec{D} = \rho$$

$$\textbf{2.} \ \nabla \bullet \vec{B} = 0$$

$$\textbf{3.} \ \nabla x \vec{H} = \vec{J} \ \textbf{(Amperes law)}$$

$$\textbf{4.} \ \nabla x \vec{E} = 0$$

The following are the four Maxwell's equations for **time varying fields**.

$$1. \nabla \bullet \vec{D} = \rho$$

$$2. \ \nabla \bullet \vec{B} = 0$$

$$3. \ \nabla x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \textbf{(Modified amperes law)}$$

$$4. \ \nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \textbf{(Faradays law)}$$

EQUATION OF CONTINUITY:

Consider a closed surface enclosing a volume of charge density ρ. If there is any charge flow through this surface, then charge flow through a small area ds is $\vec{J} \bullet \vec{ds}$

$$\text{Total amount of charge flowing outwards} = \int_s \vec{J} \bullet \vec{ds}$$

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Consider a small volume dv inside the surface , the charge in the volume dv = ρ dv

Charge in the entire volume $\quad = \int_{v} \rho \ dv$

As the charge flows outwards, the net charge present inside the volume decreases with time.

Therefore, amount of charge flowing outwards = Decrease in the net charge inside the volume

$$\int_{s} \vec{J} \bullet \vec{ds} \ = - \ \frac{\partial}{\partial t} \int_{v} \rho \ dv \quad \text{[The negative sign represents the decrease in charge]}$$

$$\Rightarrow \int_{V} \nabla \bullet \vec{J} \ dv = -\frac{\partial}{\partial t} \int_{V} \rho dv$$

$$\Rightarrow \nabla \bullet \vec{J} = -\frac{\partial \rho}{\partial t}$$

Maxwell's fixed amperes law

Consider the expression for amperes law ,

$$\nabla x \vec{H} = \vec{J}$$

Taking divergence on both sides

$$\nabla \bullet (\nabla x \vec{H}) = \nabla \bullet \vec{J}$$

Since the divergence of a curl is zero

$$\nabla \bullet \vec{J} = 0$$

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This means that divergence of current is zero or there is no net accumulation or loss of charge.[There are no sources or sinks of current].This is true in the case of steady ,direct currents which are continuous through out the circuit.

However in a circuit containing a capacitor, during discharging, the current starts at the positively charged plate and the charge diminishes as the current flows to the negatively charged plate where it accumulates. Hence we can look upon the condenser plates as sources or sinks of currents. So Maxwell concluded that the equation $\nabla \bullet \vec{H} = \vec{J}$ is incomplete and besides the term $\vec{J}$, there must be a term added such that the sum of the two will give zero divergence.

The new term was found as follows.

From the equation of continuity

$$\nabla \bullet \vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{where } \rho \text{ is the charge density}$$

$$\nabla \bullet \vec{J} = -\frac{\partial}{\partial t}\left(\nabla \bullet \vec{D}\right) = \nabla \bullet \frac{\partial \vec{D}}{dt}$$

$$\Rightarrow \nabla \bullet \left(\vec{J} + \frac{\partial \vec{D}}{dt}\right) = 0$$

Thus although $\nabla \bullet \vec{J}$ is not zero but the divergence of $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$ is always zero .Hence Maxwell made the assumption that the term $\vec{J}$ in amperes law must be replaced by $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$.

$$\therefore \textbf{For time varying fields} \quad \nabla x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

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The term $\left(\dfrac{\partial \vec{D}}{\partial t} \right)$ is known as displacement current density .It is clear from the above expression that

the rate of change of electric displacement produces magnetic field just as the conduction current ($\vec{J}$

)does.

Maxwell's equations in integral form:

$$\int_S \vec{D} \bullet \vec{ds} = \int_v \rho \ dv$$

$$\int_S \vec{B} \bullet \vec{ds} = 0$$

$$\int_l \vec{H} \bullet \vec{dl} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \bullet \vec{ds}$$

$$\int_l E \bullet dl = -\int_S \frac{\partial \vec{B}}{\partial t} \bullet \vec{ds}$$

Maxwell's equations in word form:

1. The total electric flux density through a closed surface enclosing a volume V is equal to the total charge within the volume.

2. The total magnetic flux emerging through any closed surface is zero.

3. The magneto motive force around a closed path is equal to the conduction current $(\vec{J})$ plus the

rate of change of electric displacement $(\vec{D})$ through the surface bounded by the path.

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4. The electromotive force around a closed path is equal to the rate of change of magnetic flux
   through the surface bounded by the path.

ELECTROMAGNETIC WAVES

Wave equation for electric field:

Consider the equation $\quad \nabla x \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

$$= -\mu \dfrac{\partial \vec{H}}{\partial t} \qquad\qquad \therefore \vec{B} = \mu \vec{H}$$

Taking curl on both sides

$$\nabla x (\nabla x \vec{E}) = -\mu \dfrac{\partial}{\partial t}\left(\nabla x \vec{H}\right) \qquad\qquad \dots\dots\dots\dots(1)$$

Also $\nabla x (\nabla x \vec{E}) = \nabla\left(\nabla \bullet \vec{E}\right) - \nabla^2 \vec{E} = \nabla\left(\dfrac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E} \qquad\qquad \dots\dots\dots\dots(2)$

From (1) and (2) ,

$$\nabla\left(\dfrac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E} = -\mu \dfrac{\partial}{\partial t}\left(\nabla x \vec{H}\right)$$

$$\because \nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \varepsilon \vec{E} = \rho$$

$$\nabla \bullet \vec{E} = \dfrac{\rho}{\varepsilon}$$

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$$\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E} = -\mu\frac{\partial}{\partial t}\left(\vec{J} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla^2 \vec{E} - \mu\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2} = \mu\frac{\partial J}{\partial t} + \nabla\left(\frac{\rho}{\varepsilon}\right)$$

This is the characteristic form of a wave equation .The solution to this equation represents a wave

.

For a free space where there are no charges (ρ=0), no currents (J=0).

$$\nabla^2 \vec{E} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = 0 \qquad \dots\dots\dots\dots(3)$$

Wave equation for magnetic field:

Consider the equation $\nabla x \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

Taking curl on both the sides

$$\nabla x(\nabla x \vec{H}) = \nabla x \vec{J} + \varepsilon\frac{\partial}{\partial t}\left(\nabla x \vec{E}\right)$$

$$\nabla\left(\nabla \bullet H\right) - \nabla^2 H = \nabla x \vec{J} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\because \vec{D} = \varepsilon E$$

$$\nabla x E = \mu\frac{\partial H}{\partial t}$$

$$\nabla \bullet B = 0$$

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$$\nabla^2 H - \mu\varepsilon\frac{\partial^2 H}{\partial t^2} = -\nabla x \vec{J}$$

This is the characteristic form of wave equation.

For free space ,ρ = 0, J = 0,

Hence

$$\nabla^2 H - \mu\varepsilon\frac{\partial^2 H}{\partial t^2} = 0 \qquad\text{..........(4)}$$

The classical wave equation is represented as

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0 \qquad\text{...........(5)}$$

Comparing equations (3) and (4)   with (5)

We get $$V = \frac{1}{\sqrt{\mu\varepsilon}}$$

PLANE WAVES

These are the electromagnetic waves in which electric and magnetic fields are uniform over a

plane perpendicular to the direction of propagation.

For example, for an electromagnetic wave propagating along X axis , if the electric and magnetic

fields do not vary along Y and Z axes but vary along X axis ,then the wave is said to be a plane

wave.

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Ie.,
$$\frac{\partial E_y}{\partial y} = \frac{\partial E_z}{\partial z} = \frac{\partial H_y}{\partial y} = \frac{\partial H_z}{\partial z} = 0$$

Relation between E and H for a uniform plane wave :

Consider an electromagnetic plane wave propagating in X direction in free space

$$\nabla x H = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = -\frac{\partial H_z}{\partial x}\hat{y} + \frac{\partial H_y}{\partial x}\hat{z} \qquad \ldots\ldots\ldots(4)$$

Also

$$\nabla x H = \varepsilon\frac{\partial E_x}{\partial t}\hat{x} + \mu\frac{\partial E_y}{\partial t}\hat{y} + \mu\frac{\partial E_z}{\partial t}\hat{z} \qquad \ldots\ldots(5)$$

So from (4) and (5), equating their corresponding components

$$-\frac{\partial H_z}{\partial x}\hat{y} + \frac{\partial H_y}{\partial x}\hat{z} = \varepsilon\frac{\partial E_Y}{\partial t}\hat{y} + \varepsilon\frac{\partial H_z}{\partial t}\hat{z}$$

$$\Rightarrow \frac{\partial H_z}{\partial x} = -\varepsilon\frac{\partial E_y}{\partial t} \qquad\qquad\qquad \Big\}\ldots\ldots\ldots(6)$$

$$\frac{\partial H_y}{\partial x} = \varepsilon\frac{\partial E_z}{\partial t}$$

Now consider $\qquad \dfrac{\partial H_z}{\partial x} = -\varepsilon\dfrac{\partial E_y}{\partial t}$

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$$\frac{\partial H_z}{\partial x} = -\varepsilon \frac{\partial E_y}{\partial x}\left(\frac{\partial x}{\partial t}\right) = -\varepsilon v\left(\frac{\partial E_y}{\partial x}\right)$$

$$H_x = -\varepsilon v E_y = -\sqrt{\frac{\varepsilon}{\mu}}E_y$$

Similarly $$H_Y = \sqrt{\frac{\varepsilon}{\mu}}E_Z$$

So the resultant magnetic field $|H| = \sqrt{H_Y^2 + H_Z^2}$

In the same way, expression for electric field can be derived using the expression

$$\therefore \nabla x E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial E_x}{\partial x}\hat{y} + \frac{\partial E_y}{\partial x}\hat{z} \qquad \ldots\ldots\ldots(1)$$

Also

$$\nabla x E = \mu\frac{\partial H_x}{\partial t}\hat{x} + \mu\frac{\partial H_y}{\partial t}\hat{y} + \mu\frac{\partial H_z}{\partial t}\hat{z} \quad \ldots\ldots.(2)$$

So from (1) and (2), equating their corresponding components

$$-\frac{\partial E_x}{\partial x}\hat{y} + \frac{\partial E_y}{\partial x}\hat{z} = -\mu\frac{\partial H_Y}{\partial t}\hat{y} - \mu\frac{\partial H_z}{\partial t}\hat{z}$$

$$\Rightarrow \frac{\partial E_x}{\partial x} = \mu\frac{\partial H_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = -\mu\frac{\partial H_z}{\partial t}$$

$\left. \right\}\ldots\ldots\ldots(3)$

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It can be shown that

$$E_Z = -\sqrt{\frac{\mu}{\varepsilon}}H_Y$$

$$E_Y = \sqrt{\frac{\mu}{\varepsilon}}H_Z$$

$$|E| = \sqrt{E_Y^2 + E_Z^2}$$

Hence $\left|\dfrac{E}{H}\right| = \sqrt{\dfrac{\mu}{\varepsilon}}$

Note: To show that E and H are perpendicular:

$$E \bullet H = (E_y\hat{y} + E_z\hat{z})(H_y\hat{y} + H_z\hat{z}) = E_yH_y + E_zH_z = E_y\left(-\sqrt{\frac{\varepsilon}{\mu}}\right)E_z + E_z\left(\frac{\varepsilon}{\mu}\right)E_y = 0$$

Hence E and H are at right angles to each other.

TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES:

Consider a uniform plane wave propagating along X direction in a medium where there are no free charges.

From Maxwell's equation $\nabla.\vec{D} = \rho = 0$

$$\nabla.\varepsilon\vec{E} = \rho = 0$$

For free space

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \qquad \text{..........(1)}$$

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As the wave is propagating along X direction .So E_y and E_z remain constant along Y and Z direction respectively.

$$\frac{\partial E_y}{\partial y} = 0 = \frac{\partial E_z}{\partial z}$$

So from (1) $\frac{\partial E_x}{\partial x} = 0$ …..(2)

$\therefore \frac{\partial E_x}{\partial x} = 0 \Rightarrow E_x = \text{Constant or } E_x = 0.$

For a plane wave E_x cannot be a constant as it is periodic.

$$\therefore E_x = 0$$

This means that a uniform plane wave progressing in the x direction has no X component of electric field. It indicates that electric field is perpendicular to the direction of propagation.

A similar analysis would show that the X component of magnetic field is also zero.

POLARIZATION OF PLANE WAVE:

The polarization of a uniform plane wave refers to the time varying behavior of the electric field at some point in space.

Linear polarization:

In a linearly polarized electromagnetic wave, the electric vector remains in one direction at all times.

For example, consider a wave propagating along Z direction .

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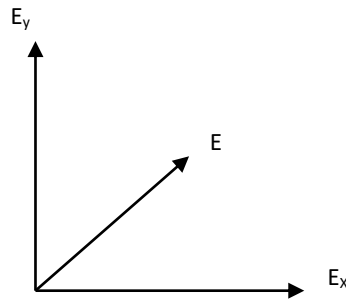
The electric field can have X and Y components.The resultant electric field vector will be oriented in a particular direction $\tan\theta = E_y / E_X$ .As the wave advances E_X and E_y change in magnitude but the change is always such that the ratio E_Y / E_X remains constant. The electric vector varies from 0 to E_{max} and back to zero in one direction and then to 0 to E_{max} and back to zero in opposite direction. During one cycle, the electric vector appears to traverse a straight line as shown in figure .

The electric intensity of a uniform plane wave in a conducting medium can be written as

$$\vec{E}(x,t) = E_0 \, e^{-\alpha z} \cos(wt - \beta z)\hat{a}_x$$

This represents a linearly polarized wave as the electric field is always in the x direction .

This can assumed to be two components

$$\vec{E}_x(z,t) = E_{0_x}e^{-\alpha z} \cos(wt - \beta z + \theta_x)$$

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MODULE -3   **SUBJECT CODE: - 18 PHY 12 /22**
(Maxwell's equations, EM Waves and Optical fibers)
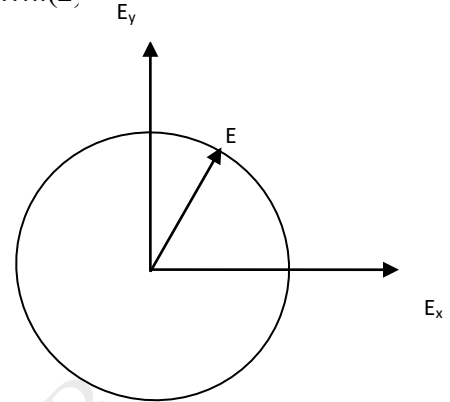
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$$\vec{E}_y(z,t) = E_{oy}e^{-\alpha z}\cos(wt - \beta z + \theta_y)$$

At z =0 and when they are in phase , $\theta_x = \theta_y = \theta$

Then $\vec{E}_x(0,t) = E_{0_x}\cos(wt - \beta z + \theta)$

$\vec{E}_y(0,t) = E_{oy}\cos(wt - \beta z + \theta)$

So $\vec{E}_x(0,t) = \dfrac{E_{ox}}{E_{oy}}\ E_y(0,t)$

This equation describes a linear relationship between two components. Linear polarization is a special case of elliptical polarization in which the phase difference is zero or 180.

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Elliptical Polarization:

In an elliptically polarized plane wave the electric vector E constantly changes both its magnitude and direction and electric vector describes an ellipse during one cycle.

$$\frac{E^2{}_x}{} + \frac{E_Y^2}{} = 1$$
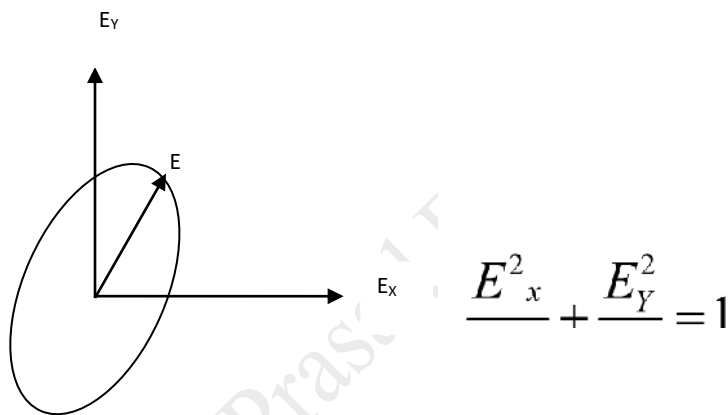
In an elliptically polarized wave , there will be a phase difference (not equal to 90^0)between E_x and

E_y .When the components are out of phase , the electric vector E rotates as a function of time describing an ellipse. It attains the maximum value E_{max} in the direction of major axis and a minimum value E_{min} in the direction of minor axis.

$$\vec{E}_x(z,t) = E_{0_x}e^{-\alpha z}\cos(wt - \beta z + \theta_x)$$

$$\vec{E}_y(z,t) = E_{oy}e^{-\alpha z}\cos(wt - \beta z + \theta_y)$$

At z =0, $\theta_x = \theta_y \pm 90^0$ and $E_{ox} \neq E_{oy}$

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$$\frac{\vec{E}_X(0,t)}{E_{0x}} = \cos(wt - \beta z + \theta_x)$$

$$\frac{\vec{E}_y(0,t)}{E_{0y}} = \sin(wt - \beta z + \theta_x)$$

$$\frac{\vec{E^2}_x(0,t)}{E^2_{0x}} + \frac{\vec{E}_y^2(0,t)}{E^2_{0y}} = 1$$

Circular polarization:

In a circularly polarized wave, the electric vector describes a circle over one cycle as the plane wave advances.   Here the phase difference between the X and Y components of electric field will be out of phase by 90 .The electric vector will have a constant magnitude but changes its direction describing a circle.

$$\vec{E}_x(x,t) = E_{0x}e^{-\alpha z}\cos(wt - \beta z + \theta_x)$$

$$\vec{E}_y(x,t) = E_{oy}e^{-\alpha z}\cos(wt - \beta z + \theta_y)$$

When $E_{ox} = E_{oy} = E_o$ and at z =0, $\theta_x = \theta_y \pm 90^0$

$$\vec{E}_x(z,t) = E_0\, e^{-\alpha z}\cos(wt - \beta z + \theta_x) \ldots\ldots\ldots\ldots(1)$$

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$$\vec{E}_y(z,t) = E_o e^{-\alpha z} \cos(wt - \beta z + \theta_x) \quad ……………(2)$$

squaring and adding (1) and (2)

$$\vec{E^2}_x(0,t) + \vec{E^2}_Y(0,t) = E^2_0$$

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Optical Fibers

Total Internal Reflection:

When a ray of light travels from denser to rarer medium it bends away from the normal. As the angle of incidence increases in the denser medium, the angle of refraction also increases. For a particular angle of incidence called the "*critical angle*", the refracted ray grazes the surface separating the media or the angle of refraction is equal to 90°. If the angle of incidence is greater than the critical angle, the light ray is reflected back to the same medium. This is called *"Total Internal Reflection"*.

In total internal reflection, there is no loss of energy. The entire incident ray is reflected back.

XX^1 is the surface separating medium of refractive index n_1 and medium of refractive index n_2, $n_1 > n_2$.

AO and OA^1 are incident and refracted rays. θ_1 and θ_2 are angle of incidence and angle of refraction, $\theta_2 > \theta_1$. For the ray BO, θ_c is the critical angle. OB^1 is the refracted ray which grazes the interface. The ray CO incident with an angle greater than θ_c is totally reflected back along OC^1.
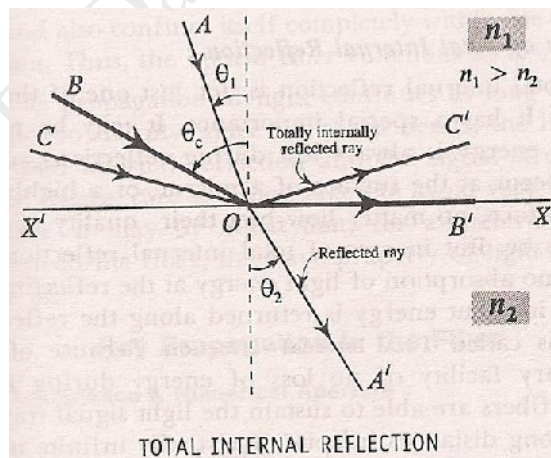
TOTAL INTERNAL REFLECTION

From Snell's law,

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$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

For total internal reflection,

$\theta_1 = \theta_c$ and $\theta_2 = 90°$

$n_1 \sin\theta_c = n_2$ (because $\sin 90° = 1$)

$\theta_c = \sin^{-1}(n_2/n_1)$

In total internal reflection there is no loss or absorption of light energy.

The entire energy is returned along the reflected light. Thus is called Total internal reflection.

Optical Fibers:

They are used in optical communication. It works on the principle of Total internal reflection(TIR).

Optical fiber is made from transparent dielectrics. It is cylindrical in shape. The inner cylindrical part is called as core of refractive index n_1. The outer part is called as cladding of refractive index n_2, $n_1 > n_2$. There is continuity between core and cladding. Cladding is enclosed inside a polyurethane jacket. Number of such fibers is grouped to form a cable.
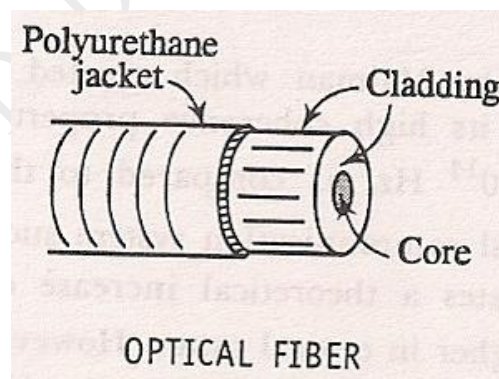
The light entering through one end of core strikes the interface of the core and cladding with angle greater than the critical angle and undergoes total internal reflection. After series of such total internal reflection, it emerges out of the core. Thus the optical fiber works as a

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waveguide. Care must be taken to avoid very sharp bends in the fiber because at sharp bends, the light ray fails to undergo total internal reflection.

FIBER AS A WAVEGUIDE
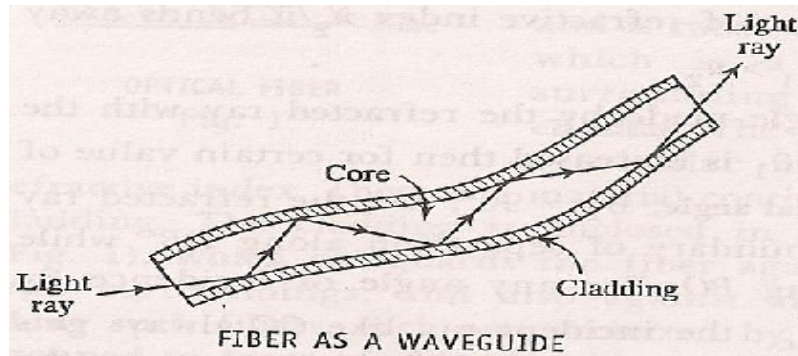
Angle of Acceptance and Numerical Aperture:

Consider a light ray AO incident at an angle 'θ_0' enters into the fiber. Let 'θ_1' be the angle of refraction for the ray OB. The refracted ray OB incident at a critical angle ($90° - \theta_1$) at B grazes the interface between core and cladding along BC. If the angle of incidence is greater than critical angle, it undergoes total internal reflection. Thus θ_0 is called the waveguide acceptance angle and $\sin\theta_0$ is called the numerical aperture.

Let n_0, n_1 and n_2 be the refractive indices of the medium, core and cladding respectively.

From Snell's law,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 \qquad\qquad \rightarrow (1)$$

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ACCEPTANCE CONE
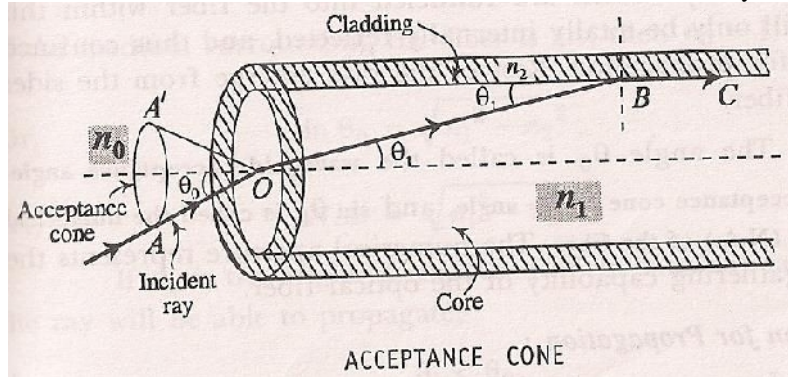
At B the angle of incidence is (90 - θ_1)

From Snell's law,

$n_1 \sin(90-\theta_1) = n_2 \sin 90$

$n_1 \cos\theta_1 = n_2$

$$\cos\theta_1 = n_2 / n_1 \qquad\qquad \rightarrow (2)$$

From eqn (1)

$$Sin\,\theta_0 = \frac{n_1}{n_0}\sin\theta_1$$

$$= \frac{n_1}{n_o}\sqrt{1 - cos^2\theta}_1 \qquad\qquad \rightarrow (3)$$

Using eqn (2) in (3)

$$Sin\,\theta_0 = \frac{n_1}{n_0}\sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$= \frac{\sqrt{n_1^2 - n_2^2}}{n_o} \qquad\qquad \rightarrow (4)$$

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The surrounding medium is air, $n_o = 1$

$$Sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

Where $\sin\theta_i$ is called numerical aperture.

$$N.A = \sqrt{n_1^2 - n_2^2}$$

Therefore for any angle of incidence equal to θ_i equal to or less than θ_0, the incident ray is able to propagate.

$$\theta_i < \theta_0$$

$$Sin\,\theta_i < Sin\,\theta_0$$

$$sin\theta_i < \sqrt{n_1^2 - n_2^2}$$

$sin\theta_i < N.A$ is the condition for propagation.

Fractional Index Change:

"It is the ratio of the refractive index difference between the core and cladding to the refractive index of the core of an optical fiber".

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Relation between N.A and Δ:

$$\text{Consider } \Delta = \frac{n_1 - n_2}{n_1}$$

$$n_1 - n_2 = \Delta n_1$$

We have

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$$\text{N.A} = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

Considering $n_1 \approx n_2$

$$= \sqrt{(n_1 + n_2)\Delta n_1}$$

$$N.A = \sqrt{2n_1^2 \Delta}$$

$$N.A = n_1\sqrt{2\Delta}$$

Increase in the value of Δ increases N.A

It enhances the light gathering capacity of the fiber. Δ value cannot be increased very much because it leads to intermodal dispersion intern signal distortion.

V-number:

The number of modes supported for propagation in the fiber is determined by a parameter called V-number.

If the surrounding medium is air, then

$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$$

Where 'd' is the core diameter, n_1 and n_2 are refractive indices of core and cladding respectively, 'λ' is the wavelength of light propagating in the fiber.

$$V = \frac{\pi d}{\lambda}(NA)$$

If the fiber is surrounded by a medium of refractive index n_0, then,

$$V = \frac{\pi d}{\lambda} \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

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For V >1, the number of modes supported by the fiber is given by, number of modes ≈ $V^2/2$. If V= 2.405 or less than 2.405, then the fibre is called single mode otherwise it is called as multimode fibre.

Types of optical fibers:

In an optical fiber the refractive index of cladding is uniform and the refractive index of core may be uniform or may vary in a particular way such that the refractive index decreases from the axis, radically.

Following are the different types of fibers:

1. Single mode fiber
2. Step index multimode fiber
3. Graded index multimode fiber

1. _Single mode fiber_: Refractive index of core and cladding has uniform value; there is an increase in refractive index from cladding to core. They are used in submarine.
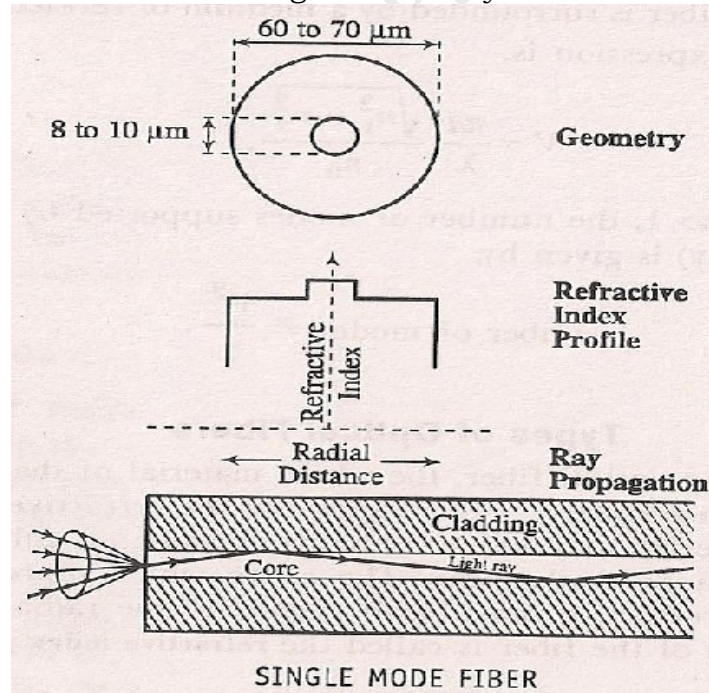
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2. _Step index multimode fiber:_ It is similar to single mode fiber but core has large diameter. It can propagate large number of modes as shown in figure. Laser or LED is used as a source of light. It has an application in data links.

STEP - INDEX MULTIMODE FIBER
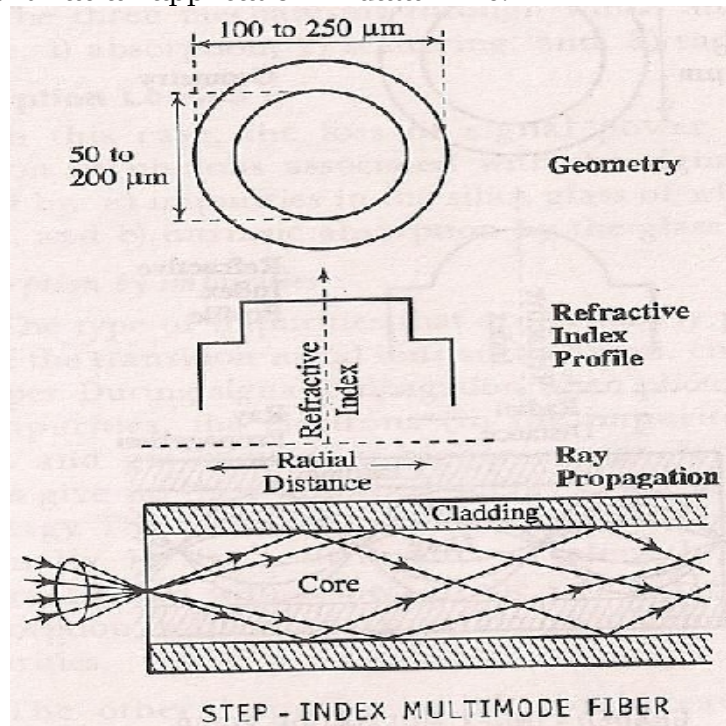
3. **_Graded index multimode fiber:_** It is also called GRIN. The refractive index of core decreases from the axis towards the core cladding interface. The refractive index profile is shown in figure. The incident rays bends and takes a periodic path along the axis. The rays have different paths with same period. Laser or LED is used as a source of light. It is the expensive of all. It is used in telephone trunk between central offices.

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GRADED - INDEX MULTIMODE FIBER
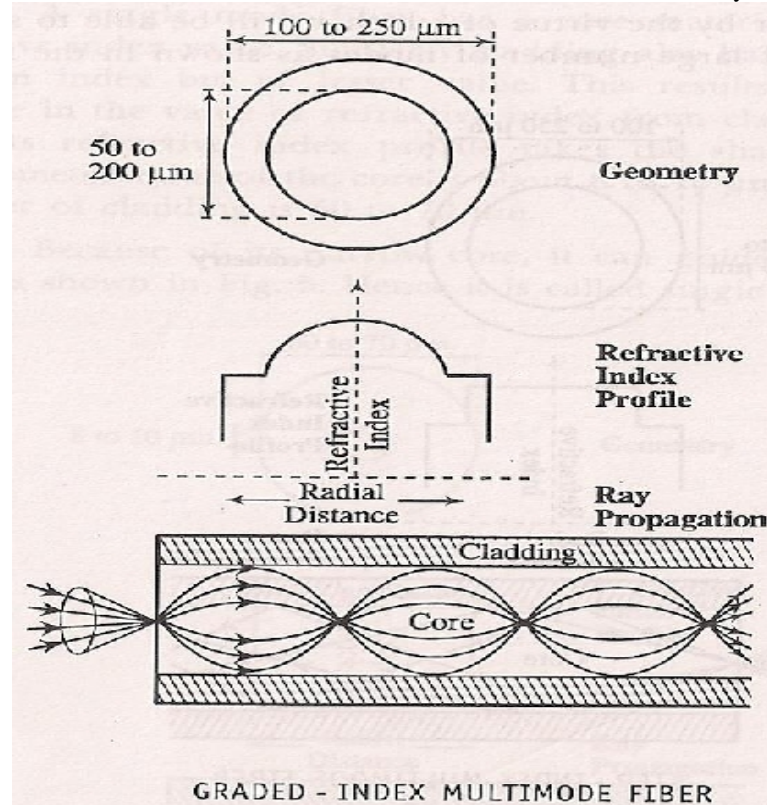
Signal distortion in optical fibers:

The propagation of a signal through the optical fiber involves total internal reflection of light rays many times. Further, the rays are reflected at various angles. The rays reflected at higher angles travel greater distances than the rays reflected at lower angles. As a result, all the rays do not arrive at the end of the fiber simultaneously and the light pulse broadens as it travels through the fiber. Since the output pulse does not match with the input pulse, the signal is said to be distorted.

If white light is used instead of monochromatic light, another kind of distortion occurs. Since radiation of different wavelengths has different velocities, they do not arrive at the output simultaneously. This distortion is called chromatic dispersion.

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The signal distortion is quite considerable in multimode step index fibers. In graded index fibers, the light travels with different velocities in different parts of the core as the refractive index varies radially along the core. The rays travel faster near the interface. Hence all the rays arrive at the output almost at the same time and the signal distortion is reduced. In a single mode step index fiber the distortion is less than that in multimode step index fibers.

Signal attenuation in optical fibers:

Attenuation is the loss of optical power as light travels through a fiber. It is expressed in decibel/kilometer [db/km]. A fiber with lower attenuation will allow more power to reach its receiver than a fiber with higher attenuation. If P_{in} is the input power and P_{out} is the output power after passing through a fiber of length 'L', the mean attenuation constant or coefficient 'α' of the fiber, in units of dB/km is given by

$$\alpha = -\frac{10}{L} \log_{10}\left(\frac{p_{out}}{p_{in}}\right) \quad \text{dB/km}$$

Attenuation can be caused by three mechanisms.

1. Absorption:- Absorption of photons by impurities like metal ions such as iron, chromium, cobalt and copper in the silica glass of which the fiber is made of. During signal processing photons interact with electrons of impurity atoms. The atoms are excited and de-excite by emitting photons of different characteristics. Hence it is a loss of energy. The other impurity such as hydroxyl ions (OH) causes significant absorption loss. The absorption of photons by fiber material itself is called intrinsic absorption.

2. Scattering: When the wavelength of the photon is comparable to the size of the particle then the scattering takes place. Because of the non uniformity in manufacturing, the refractive index changes with length leads to a scattering. This type of scattering is called as Rayleigh scattering. It is inversely proportional to the fourth power of wavelength. Scattering of photons also takes place due to trapped gas bubbles which are not dissolved at the time of manufacturing.

3. Radiation losses: Radiation losses occur due to macroscopic bends and microscopic bends.

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Macroscopic bending: All optical fibers are having critical radius of curvature provided by the manufacturer. If the fiber is bent below that specification of radius of curvature, the light ray incident on the core cladding interface will not satisfy the condition of TIR. This causes loss of optical power.

Microscopic bending: Optical power loss in optical fibers is due to non-uniformity of the optical fibers when they are laid. Non uniformity is due to manufacturing defects and also lateral pressure built up on the fiber. The defect due to non uniformity (microbendings) can be overcome by introducing optical fiber inside a good strengthen polyurethane jacket.

Fiber Optics Communication System:

```
┌──────────┐   ┌───────────┐   ┌──────────┐   ┌──────────┐
│ Message  │──>│ Modulator │──>│ Carrier  │──>│ Channel  │
│ Origin   │   │           │   │ Source   │   │ Coupler  │
└──────────┘   └───────────┘   └──────────┘   └──────────┘
                                                     │
                                                     v
                                               ┌──────────┐
                                               │ Optical  │
                                               │ Fibers   │
                                               └──────────┘
                                                     │
                                                     v
┌──────────┐   ┌───────────┐   ┌──────────┐   ┌──────────┐
│ Message  │<──│ Signal    │<──│ Detector │<──│ Channel  │
│ output   │   │ Processor │   │          │   │ Coupler  │
└──────────┘   └───────────┘   └──────────┘   └──────────┘
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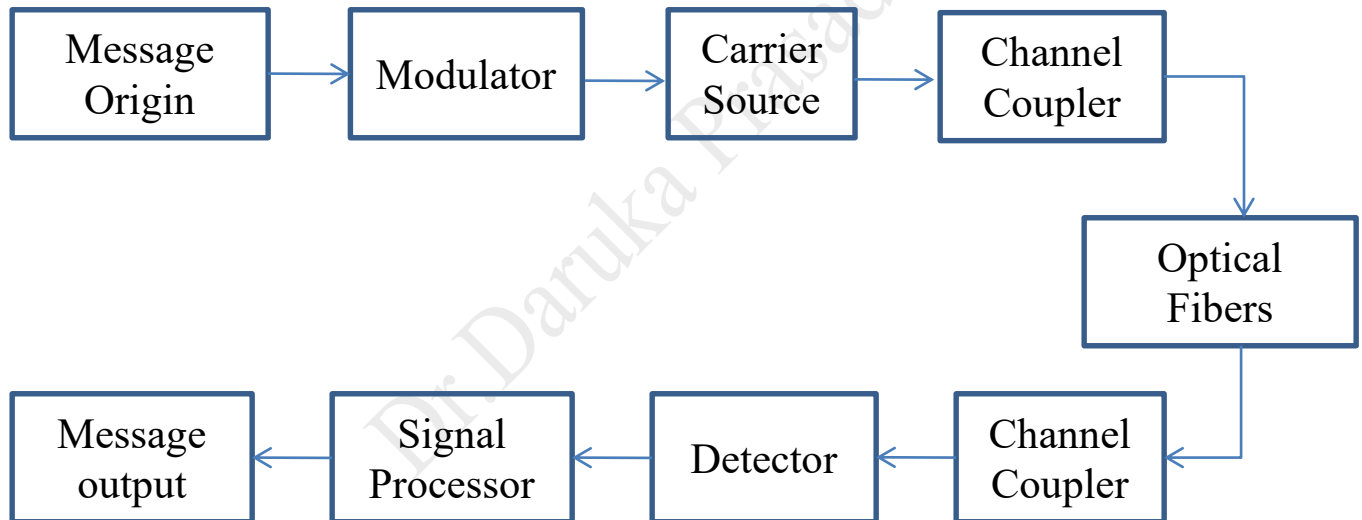
Optical fiber communication system consists of transmitter, information channel and receiver. Transmitter converts an electrical signal into optical signal. Information channel carries the signal from transmitter to receiver. The receiver converts optical signal to electrical form. The block diagram of optical fiber communication system is shown in fig.

Message origin: It converts a non electrical message into an electrical signal.

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Modulator: It converts the electrical message into proper format and it helps to improve the signal onto the wave which is generated by the carrier source.

There are two types of format. They are Analog and digital. Analog signal is continuous and it doesn't make any change in the original format. But digital signal will be either in ON or OFF state.

Carrier source: It generates the waves on which the data is transmitted. These carrier waves are produced by the electrical oscillator. Light emitting diodes (LED) and laser diodes (LD) are the different sources.

Channel Coupler: (Input) The function of the channel coupler is to provide the information to information channel. It can be an antenna which transfers all the data.

Information channel: It is path between transmitter and receiver. There are two types of information channel. They are guided and unguided. Atmosphere is the good example for unguided information channel. Co-axial cable, two-wire line and rectangular wave guide are example for guided channel.

Channel Coupler: (Output) The output coupler guides the emerged light from the fiber on to the light detector.

Detector: The detector separates the information from the carrier wave. Here a photo-detector converts optical signal to electronic signal.

Signal processor: Signal processor amplifies the signals and filters the undesired frequencies.

Message output: The output message will be in two forms. Either person can see the information or hear the information. The electrical signal can be converted into sound wave or visual image by using CRO.

Advantages of optical communication system:

1) It carries very large amount of information in either digital or analog form due to its large bandwidth.
2) The materials used for making optical fiber are dielectric nature. So, it doesn't produces or receives any electromagnetic and R-F interferences.
3) Fibers are much easier to transport because of their compactness and lightweight.
4) It is easily compatible with electronic system.

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5) It can be operated in high temperature range.
6) It does not pick up any conducted noise.
7) Not affected by corrosion and moisture.
8) It does not get affected by nuclear radiations.
9) No sparks are generated because the signal is optical signal.

- Note: - Optical fibers are used in sensors like pressure sensor, voltage sensor and current sensors.

    Optical fibers are used in local networks like data link purpose.
    --------_____-----

1) Optical fibers work on the principle of ---- total internal reflection
2) If n_1 is the refractive index of the core and n_2 is the refractive index of the cladding then for total internal reflection at the core and cladding
   Interface---- $n_1 > n_2$
3) n_1 and n_2 are the refractive indices of the core and cladding respectively and if θ_c is the critical angle then ------ $\theta_c = \sin^{-1} \frac{n_2}{n_1}$
4) if n_1, n_2, n_0 are the refractive indices of the core cladding and the surrounding medium respectively then the waveguide angle θ_o is $\sin \theta_o = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_o}$
5) If θ_i is the angle of incidence in an optical fiber and N.A is the numerical aperture then the condition for the propagation is------ $\sin \theta_i$ < N.A
6) Numerical aperture for the optical fiber depends upon the -----acceptance angle.
7) If n_1 and n_2 the are refractive indices of core and cladding then the fractional index change is given by----------- $\Delta = \frac{(n_1 - n_2)}{n_1}$
8) Fractional index change for an optical fiber with core and cladding refractive indices 1.563 and 1.498 respectively is   ------- 0.04159
9) Number of mode supported by optical fiber cable is -----v $= \frac{\pi d}{\lambda} \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}$
10) For v>> 1 the number of modes supported by the fiber cable is ------ $\approx \frac{V^2}{2}$

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(Maxwell's equations, EM Waves and Optical fibers)

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Engineering Physics 2018-19 CBCS Scheme   Name of the Faculty:- Dr. Daruka Prasad B

11) Loss of power by an optical signal through the optical fiber is mainly due to Rayleigh scattering.

12) In an optical fiber Rayleigh scattering occurs when a photon ----- encounters sharp change in the refractive index over distance smaller than its wavelength

13) Attenuation means ----- loss of signal strength

14) Delay distortion occurs due to ------- spreading of pulse

15) Having cladding around the core is preferred to coating the core with a reflecting material (silvering) because ----- reflection at the core and cladding interface is superior to the one by any coated material

16) Loss of power during transmission through optical fiber is called ------attenuation.

17) In a single mode fiber the diameter of the core is 8-10μm.

18) The numerical aperture of an optical fiber is 0.2 when surrounded by air. The acceptance angle when the fiber is in water of refractive index 1.33 is 8.65^0.

19) Attenuation in optical fiber causes due to absorption, scattering and dispersion.

20) The numerical aperture of an optical fiber in air is 0.32. The numerical aperture in water (n=4/3) is --------0.24.

21) Graded index fiber can be multimode fiber only.

22) The optical fiber kept in the medium of refractive index μ>1 instead of air, the acceptance angel is decreases.

23) In graded index fiber, the RI of cladding is constant and RI of core varies linearly from axis of fiber to the core cladding interface.

Numericals

1. An optical glass fiber of refractive index 1.50 is to be clad with another glass to ensure internal reflection that will contain light travelling within 5^{0}of the fiber axis. What maximum index of refraction is allowed for the cladding?(Jan 2010)

2. The angel of acceptance of an optical fiber is 30^0 when kept in air. Find the angle of acceptance when it is in a medium of RI 1.33 (July 2011)

3. An optical signal has lost 85% its power after travelling 400 m of fiber. What is the fiber loss? (July 2007)

4. The angle of acceptance of an optical fiber is 30^0 when kept in air. Find the angle of acceptance when it is in a medium of refractive index 1.33 (Jan 2000)

5. Calculate the numerical aperture, fractional index change and V-number for a fiber of core diameter 40μm and with refractive indices of 1.55 and 1.50 respectively for core and cladding. The wavelength of the propagating wave is 1400nm. Assume that the fiber is in air? (Jan 2008)

Descriptive type questions:

B M S INSTITUTE OF TECHNOLOGY AND MANAGEMENT
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DEPARTMENT OF PHYSICS

COURSE MATERIAL   **SUBJECT: - Engineering Physics**
MODULE -3   **SUBJECT CODE: - 18 PHY 12 /22**
(Maxwell's equations, EM Waves and Optical fibers)

Study material

Engineering Physics 2018-19 CBCS Scheme   Name of the Faculty:- Dr. Daruka Prasad B

1. What is meant by acceptance angle for an optical fiber? Show how it is related to numerical aperture. (July 2007)
2. Obtain an expression for numerical aperture and arrive the condition for propagation (Jan 2009)
3. Discuss types of optical fibers and modes of propagation using suitable diagram. (July 2007,July 2011)
4. Describe the point to point communication system, with the help of a block diagram? (Jan 2010)

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(Maxwell's equations, EM Waves and Optical fibers)

Study material

Engineering Physics 2018-19 CBCS Scheme   Name of the Faculty:- Dr. Daruka Prasad B

B. M. S. Institute of Technology, Bangalore

Course Material

Name of the Faculty : Dr. Daruka Prasad B

Department : Physics

Designation : Assistant Professor

Subject : Engineering Physics

Subject Code : 18 PHY 12 /22

B. M. S. Institute of Technology and Management
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DEPARTMENT OF PHYSICS
COURSE MATERIAL   SUBJECT: - Engineering Physics
MODULE-IV   SUBJECT CODE: - 18 PHY 12 /22

Syllabus

Quantum mechanics: Introduction to Quantum mechanics, Wave nature of particles, Heisenberg's uncertainty principle and applications (non confinement of electron in the nucleus), Schrodinger time independent wave equation, Significance of Wave function, Normalization, Particle in a box, Energy eigen values of a particle in a box and probability densities

Lasers: Review of spontaneous and stimulated processes, Einstein's coefficients (derivation of expression for energy density). Requisites of a Laser system. Conditions for laser action. Principle, Construction and working of CO_2 and semiconductor Lasers.

Application of Lasers in Defense (Laser range finder) and Engineering (Data storage)   `

Numerical problems

Quantum mechanics
Introduction to Quantum Mechanics
Blackbody Radiation spectrum:

A Blackbody is one which absorbs the entire radiation incident on it and emits all the absorbed radiation when it is more hot. A true blackbody does not exist practically. A blackbody designed by Wein has features very close to the true blackbody. Ferry has also constructed blackbody called Ferry's blackbody.   A blackbody at a particular temperature found to emit a radiation of all possible wavelengths. It is a continuous spectrum starting from certain minimum wavelength to maximum wavelength. The maximum intensity corresponds to a particular wavelength. For different temperatures of the black body, there are different curves.   As the temperature of the body increases, the wavelength corresponding to maximum intensity shifts towards lower wavelength side. The distribution of energy in black body radiation is shown in the following fig.
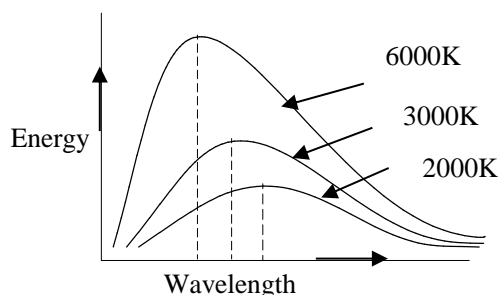
Wein's, Rayleigh-Jeans and Planck have given their explanations to account these observed experimental facts but among these only Planck's law successfully explained the total results of this experimental observations.

1. *Planck's Law:*

Planck assumed that walls of the experimental blackbody consists larger number of electrical oscillators. Each oscillator vibrates with its own frequency.

i) Each oscillator has an energy given by integral multiple of $h\upsilon$ where h is Planck's constant & υ is the frequency of vibration.

$$E = nh\upsilon = nhc/\lambda; \text{ where } n = 1, 2, 3 \ldots \text{etc.}$$

ii) An oscillator may lose or gain energy by emitting or absorbing respectively a radiation of frequency υ where $\upsilon = \Delta E/h$, ΔE is difference in energies of the oscillator before and after the emission or absorption take place.

Planck derived the law which holds good for the entire spectrum of the blackbody radiation as

$$U_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{hc/\lambda kt}-1}\right]d\lambda \qquad (\text{since } \upsilon = c/\lambda) \qquad \longrightarrow (1)$$

This is Planck's Radiation Law.

Compton Effect:

The scattering of a photon by an electron is called as Compton effect or Compton scattering.

When a photon of wavelength 'λ' is scattered by an electron in the direction making an angle 'θ' with the direction of incidence, the wavelength of the scattered photon increases. Its wavelength is λ'. The electron recoils in a direction making an angle 'φ' with the incident direction of photon. The difference in the wavelength $(\lambda'-\lambda)$ is called the Compton shift. Compton found that λ' is independent of the target material, but depends on the angle of scattering.
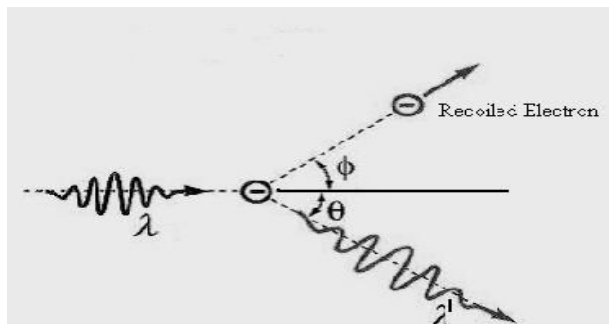
If λ is the wavelength of the incident photon, its energy E is given by $E=hc/\lambda$ where 'h' is the Planck's constant, 'c' is the velocity of light, 'λ' is the wavelength of the incident photon. If λ' is the wavelength of the scattered photon, its energy E' is given by

$$E' = hc/\lambda'$$

The energy of the scattered photon is reduced from E to E'. The difference of energy is carried by recoiling electron at an angle 'φ' with the incident direction of photon.

Applying the laws of conservation of energy and conservation of momentum Compton obtained an expression for change in wavelength $\Delta\lambda$ given by $\Delta\lambda$ $= \lambda' - \lambda = \dfrac{h}{mc}(1 - \cos\theta)$

Where 'm' is the mass of the electron, h/mc is called as Compton wavelength.

Compton Effect explains particle nature of light.

Dual nature of matter (de-Broglie Hypothesis)

Light exhibits the phenomenon of interference, diffraction, photoelectric effect and Compton Effect. The phenomenon of interference, diffraction can only be explained with the concept that light travels in the form of waves. The phenomenon of photoelectric effect and Compton Effect can only be explained with the concept of *Quantum theory of light*. It means to say that light possess particle nature. Hence it is concluded that light exhibits dual nature namely wave nature as well as particle nature.

de-Broglie's Wavelength:

A particle of mass 'm' moving with velocity 'c' possess energy given by

$E = mc^2 \qquad \rightarrow$ (Einstein's Equation) (1)

According to Planck's quantum theory the energy of quantum of frequency 'υ' is

$E = h\upsilon \qquad \rightarrow$ (2)

From (1) & (2)

$mc^2 = h\upsilon = hc/\lambda \qquad$ since $\upsilon = c/\lambda$

$\lambda = hc/mc^2 = h/mc$

$\lambda = h/mv \qquad$ since $v \approx c$

Relation between de-Broglie wavelength and kinetic energy

Consider an electron in an electric potential V, the energy acquired is given by

$$E = eV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Where 'm' is the mass, 'v' is the velocity and 'p' is the momentum of the particle. 'e' is charge of an electron.

$$p = \sqrt{2meV} = \sqrt{2mE}$$

The expression for de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}}$$

de-Broglie Wavelength of a charged particles like electron / proton etc.,:-

An electron accelerated with potential difference 'V' has energy 'eV'. If 'm' is the mass and 'v' is the velocity of the electron.

Then eV = $1/2(mv^2)$ $\rightarrow$ (1)

If 'p' is the momentum of the electron, then p=mv

Squaring on both sides, we have

p² = m²v²

mv² = p²/m

Using in equation (1) we have

eV = p²/(2m) or p=√(2meV)

According to de-Broglie λ = h/p

Therefore $\lambda = \left[\dfrac{h}{\sqrt{2meV}}\right] = \dfrac{1}{\sqrt{V}}\left[\dfrac{h}{\sqrt{2me}}\right]$

For electron

$$\lambda = \frac{1}{\sqrt{V}}\left[\frac{6.626\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.602\times10^{-19}}}\right]$$

$$= \frac{1.226\times10^{-9}}{\sqrt{V}}\,\text{m}$$

$$\boxed{\lambda = \frac{1.226}{\sqrt{V}}\,\text{nm}}$$

de-Broglie Wavelength of a neutral particle like neutron:-

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}}$$

Where E for neutral particles can be treated as similar to kinetic theory of gas molecules given by $E = \frac{3}{2}K_B T$; where K_B is Boltzmann constant = 1.38x10^{-23} J/K and T is the temperature. By substituting this in the above equation then

$$\lambda = \frac{h}{\sqrt{3mk_B T}}$$

Characteristics of matter waves:

1. Waves associated with moving particles are called matter waves. The wavelength 'λ' of a de-Broglie wave associated with particle of mass 'm' moving with velocity 'v' is
   $$\lambda = h/(mv)$$

2. Matter waves are not electromagnetic waves because the de Broglie wavelength is independent of charge of the moving particle.
3. The velocity of matter waves (v_P) is not constant. The wavelength is inversely proportional to the velocity of the moving particle.
4. Lighter the particle, longer will be the wavelength of the matter waves, velocity being constant.
5. For a particle at rest the wavelength associated with it becomes infinite. This shows that only moving particle produces the matter waves.

Phase velocity and group velocity:

A wave is represented by the equation:

$$y = A\sin(\omega t - kx)$$

Where 'y' is the displacement along Y-axis at an instant t, 'ω' is the angular frequency, 'k' is propagation constant or wave number. 'x' is the displacement along x-axis at the instant 't'.
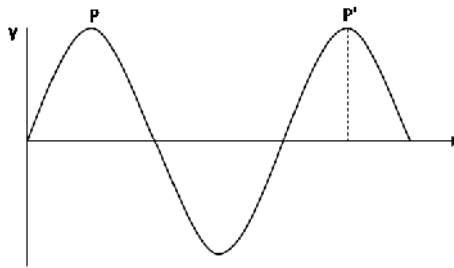
If 'p' is the point on a progressive wave, then it is the representative point for a particular phase of the wave, the velocity with which it is propagated owing to the motion of the wave is called *phase velocity.*

The phase velocity of a wave is given by $v_{phase} = (\omega / k)$.

Group velocity:
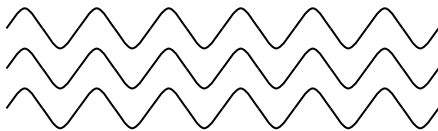
Individual Waves	*Amplitude variation after Superposition*

A group of two or more waves, slightly differing in wavelengths are super imposed on each other. The resultant wave is a packet or wave group. The velocity with which the envelope enclosing a wave group is transported is called *Group Velocity*.

$$v_{group} = \frac{(\Delta\omega/2)}{(\Delta k/2)} = \frac{\Delta\omega}{\Delta k}$$

In the limit $\dfrac{\Delta\omega}{\Delta k} \rightarrow \dfrac{d\omega}{dk}$

$$v_{group} = \frac{d\omega}{dk}$$

Quantum Mechanics

Difference between Classical Mechanics and Quantum Mechanics

According to Classical mechanics it is unconditionally accepted that the position, mass, velocity, acceleration etc., of a particle or a body can be measured accurately, which is true as we observe in every day. The values predicted by classical mechanics fully agree with measured values.

Quantum mechanics has been built upon with purely probabilistic in nature. The fundamental assumption of Quantum mechanics is that it is impossible to measure

simultaneously the position and momentum of a particle. In quantum mechanics the measurements are purely probable. For example the radius of the first allowed orbit of electron in hydrogen atom is precisely 5.3×10^{-11}m. If a suitable experiment is conducted to measure the radius, number of values are obtained which are very close to 5.3×10^{-11}m. This type of uncertainty makes classical mechanics superior to quantum mechanics. The accurate values declared by classical mechanics are found to be true in day to day life. But in the domain of nucleus, atoms, molecules etc., the probabilities involved in the values of various physical quantities become insignificant and classical mechanics fails to account such problems.

Heisenberg's Uncertainty Principle:

According to classical mechanics a particle occupies a definite place in space and possesses a definite momentum. If the position and momentum of a particle is known at any instant of time, it is possible to calculate its position and momentum at any later instant of time. The path of the particle could be traced. This concept breaks down in quantum mechanics leading to Heisenberg's Uncertainty Principle according to which "It is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa.

If Δx and ΔP_x are the uncertainties in the measurement of position and momentum of the particle then the uncertainty can be written as

$$\Delta x \, . \, \Delta P_x \geq (h/4\pi)$$

In any simultaneous determination of the position and momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than $h/4\pi$.

Similarly   1) $\Delta E . \Delta t \geq h/4\pi$ 2) $\Delta L . \Delta \theta \geq h/4\pi$

Application of Uncertainty Principle:

Impossibility of existence of electrons in the atomic nucleus:

According to the theory of relativity, the energy E of a particle is: $E = mc^2 = $

$$\frac{m_o c^2}{\sqrt{1 - (v^2/c^2)}}$$

Where 'm_o' is the rest mass of the particle and 'm' is the mass when its velocity is 'v'.

$$\text{i.e. } E^2 = \frac{m_o^2 c^4}{1 - (v^2/c^2)} = \frac{m_o^2 c^6}{c^2 - v^2} \rightarrow (1)$$

If 'p' is the momentum of the particle:

$$\text{i.e. } p = mv = \frac{m_o v}{\sqrt{1 - (v^2/c^2)}}$$

$$p^2 = \frac{m_o^2 v^2 c^2}{c^2 - v^2}$$

Multiply by c^2

$$p^2 c^2 = \frac{m_o^2 v^2 c^4}{c^2 - v^2} \rightarrow (2)$$

Subtracting (2) by (1) we have

$$E^2 - p^2 c^2 = \frac{m_o^2 c^4 (c^2 - v^2)}{c^2 - v^2}$$

$$E^2 = p^2 c^2 + m_o^2 c^4 \rightarrow (3)$$

Heisenberg's uncertainty principle states that

$$\Delta x . \Delta P_x \geq \frac{h}{4\pi} \rightarrow (4)$$

The diameter of the nucleus is of the order 10^{-14}m. If an electron is to exist inside the nucleus, the uncertainty in its position Δx must not exceed 10^{-14}m.

i.e. $\Delta x \leq 10^{-14}$m

The minimum uncertainty in the momentum

$$(\Delta P_x)_{\min} \geq \frac{h}{4\pi \, (\Delta x)_{\max}} \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}} \geq 0.5 \times 10^{-20} \text{ kg. m/s} \rightarrow (5)$$

By considering minimum uncertainty in the momentum of the electron

i.e., $(\Delta P_x)_{\min} \geq 0.5 \times 10^{-20}$ kg.m/s = p $\rightarrow$ (6)

Consider eqn (3)

$$E^2 = p^2 c^2 + m_o^2 c^4 = c^2(p^2 + m_o^2 c^2)$$

Where $m_o = 9.11 \times 10^{-31}$ kg

If the electron exists in the nucleus its energy must be

$$E^2 \geq (3 \times 10^8)^2[(0.5 \times 10^{-20})^2 + (9.11 \times 10^{-31})^2(3 \times 10^8)^2]$$

i.e. $E^2 \geq (3 \times 10^8)^2[0.25 \times 10^{-40} + 7.4629 \times 10^{-44}]$

Neglecting the second term as it is smaller by more than the 3 orders of the magnitude compared to first term.

Taking square roots on both sides and simplifying

$$E \geq 1.5 \times 10^{-12} \text{ J} \geq \frac{1.5 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ ev} \geq 9.4 \text{ Mev}$$

If an electron exists in the nucleus its energy must be greater than or equal to 9.4Mev. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus.

[**Beta decay:** In beta decay process, from the nucleus of an atom, when neutrons are converting into protons in releasing an electron (beta particle) and an antineutrino. When proton is converted into a neutron in releasing a positron (beta particle) and a neutrino. In both the processes energy sharing is statistical in nature. When beta particles carry maximum energy neutrino's carries minimum energy and vice-versa. In all other processes energy sharing is in between maximum and minimum energies. The maximum energy carried by the beta particle is called as the end point energy (E_{max}).
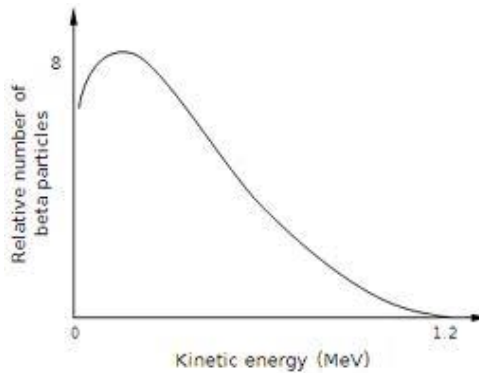
Wave Function:

A physical situation in quantum mechanics is represented by a function called wave function. It is denoted by 'ψ'. It accounts for the wave like properties of particles. Wave function is obtained by solving Schrodinger equation. To solve Schrodinger equation it is required to know

1) Potential energy of the particle

2) Initial conditions and

3) Boundary conditions.

There are two types of Schrodinger equations:

1) *The time dependent Schrodinger equation:* It takes care of both the position and time variations of the wave function. It involves imaginary quantity *i*.

The equation is: $-\dfrac{h^2}{8\pi^2 m}\dfrac{d^2\psi}{dx^2} + V\psi = -\dfrac{ih}{2\pi}\dfrac{d\psi}{dt}$

2) *The time independent Schrodinger equation*: It takes care of only position variation of the wave function.

The equation is: $\dfrac{d^2\psi}{dx^2} + \dfrac{8\pi^2 m}{h^2}(E-V)\psi = 0$

Time independent Schrodinger wave equation

Consider a particle of mass 'm' moving with velocity 'v'. The de-Broglie wavelength 'λ' is

$$\lambda = \frac{h}{m\text{v}} = \frac{h}{P} \rightarrow (1)$$

Where 'mv' is the momentum of the particle.

The wave eqn is

$$\psi = A e^{i(kx - \omega t)} \rightarrow (2)$$

Where 'A' is a constant and 'ω' is the angular frequency of the wave.

Differentiating equation (2) with respect to 't' twice

$$\frac{d^2\psi}{dt^2} = -A\omega^2 e^{i(kx-\omega t)} = -\omega^2 \psi \quad \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

Where 'y' is the displacement and 'v' is the velocity.

Similarly for the de-Broglie wave associated with the particle

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2} \frac{d^2\psi}{dt^2} \rightarrow (4)$$

where 'ψ' is the displacement at time 't'.

From eqns (3) & (4)

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi$$

But ω = 2πυ and v = υ λ where 'υ' is the frequency and 'λ' is the wavelength.

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \text{ or } \frac{1}{\lambda^2} = -\frac{1}{4\pi^2\psi}\frac{d^2\psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}mv^2 = \frac{m^2 v^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$= \frac{h^2}{2m\lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m}\left(-\frac{1}{4\pi^2\psi}\right)\frac{d^2\psi}{dx^2} = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} \rightarrow (8)$$

Total Energy E = K.E + P.E

$$E = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2}$$

$$\frac{d^2\psi}{dx^2} = -\frac{8\pi^2 m}{h^2}(E-V)\psi$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0}$$

Which is the time independent Schrodinger wave equation.

Physical significance of wave function:

Probability density: If ψ is the wave function associated with a particle, then $|\psi|^2$ is the probability of finding a particle in unit volume. If 'τ' is the volume in which the particle is present but where it is exactly present is not known. Then the probability of finding a particle in certain elemental volume $d\tau$ is given by $|\psi|^2 d\tau$. Thus $|\psi|^2$ is called probability density. The probability of finding an event is real and positive quantity. In the case of complex wave functions, the probability density is $|\psi|^2 = \psi * \psi$ where $\psi*$ is Complex conjugate of ψ.
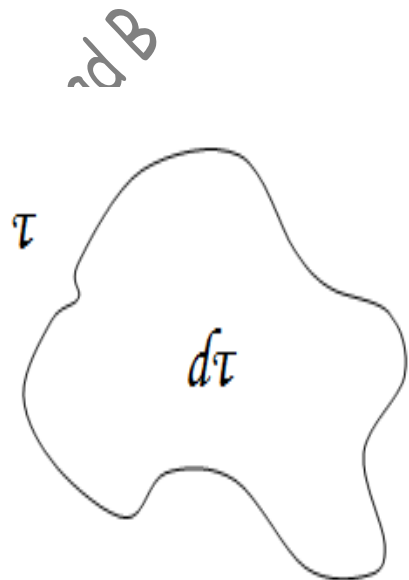
Normalization:

The probability of finding a particle having wave function 'ψ' in a volume '$d\tau$' is '$|\psi|^2 d\tau$'. If it is certain that the particle is present in finite volume 'τ', then

$$\int_0^\tau |\psi|^2 \, d\tau = 1$$

If we are not certain that the particle is present in finite volume, then

$$\int_{-\infty}^\infty |\psi|^2 d\tau = 1$$

In some cases $\int |\psi|^2 d\tau \neq 1$ and involves constant.

The process of integrating the square of the wave function within a suitable limits and equating it to unity the value of the constant involved in the wavefunction is estimated. The constant value is substituted in the wavefunction. This process is called as normalization. The wavefunction with constant value included is called as the normalized wavefunction and the value of constant is called normalization factor.

Properties of the wave function:

A system or state of the particle is defined by its energy, momentum, position etc. If the wave function 'ψ' of the system is known, the system can be defined. The wave function 'ψ' of the system changes with its state. To find 'ψ' Schrodinger equation has to be solved. As it is a second order differential equation, there are several solutions. All the solutions may not be correct. We have to select those wave functions which are suitable to the system. The acceptable wave function has to possess the following properties:

1) 'ψ' is single valued everywhere: Consider the function f(x) which varies with position as represented in the graph. The function f(x) has three values f_1, f_2 and f_3 at x = p. Since $f_1 \neq f_2 \neq f_3$ it is to state that if f(x) were to be the wave function. The probability of finding the particle has three different values at the same location which is not true. Thus the wave function is not acceptable.
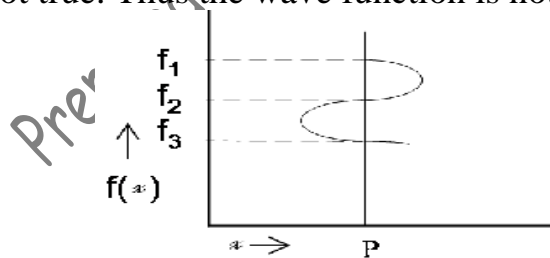
2) 'ψ' is finite everywhere: Consider the function f(x) which varies with position as represented in the graph. The function f(x) is not finite at x =R but f(x)=∞. Thus it indicates large probability of finding the particle at a location. It violates uncertainty principle. Thus the wave function is not acceptable.
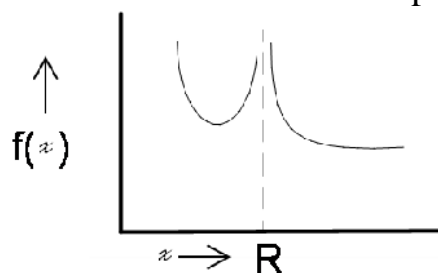
3) **'ψ' and its first derivatives with respect to its variables are continuous everywhere**: Consider the function f(x) which varies with position as represented in the graph. The function f(x) is truncated at x=Q between the points A & B, the state of the system is not defined. To obtain the wave function associated with the system, we have to solve Schrodinger wave equation. Since it is a second order differential wave equation, the wave function and its first derivative must be continuous at x=Q. As it is a discontinuous wave function, the wave function is not acceptable.
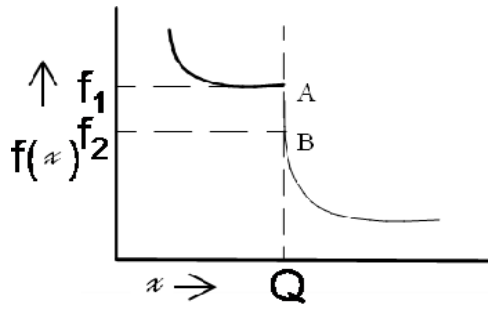
4) For bound states 'ψ' must vanish at potential boundary and outside. If 'ψ*' is a complex function, then ψ* ψ must also vanish at potential boundary and outside.

The wave function which satisfies the above 4 properties are called *Eigen functions*.

Eigen functions:

Eigen functions are those wave functions in Quantum mechanics which possesses the properties:

1. They are single valued.
2. Finite everywhere and
3. The wave functions and their first derivatives with respect to their variables are continuous.

Eigen values:

According to the Schrodinger equation there is more number of solutions. The wave functions are related to energy E. The values of energy E_n for which Schrodinger equation solved are called Eigen values.

Application of Schrodinger wave equation:

PARTICLE IN ONE-DIMENSIONAL POTENTIAL BOX:-

Energy Eigen values of a particle in one dimensional, infinite potential well (potential well of infinite depth) or of a particle in a box.
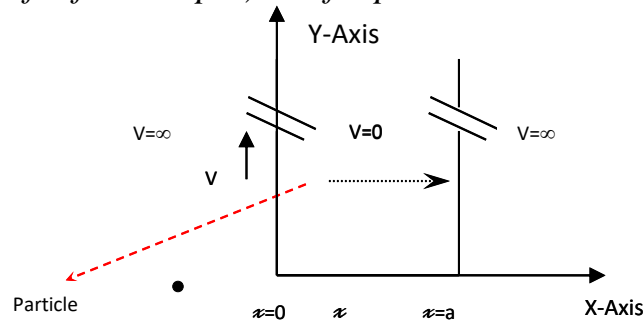
Consider a particle of a mass 'm' free to move in one dimension along positive x-direction between $x=0$ to $x=$a. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-\infty)\psi = 0 \rightarrow (1) \qquad \because V = \infty$$

For outside, the equation holds good if ψ = 0 & |ψ|² = 0. That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0 \rightarrow (2) \qquad \because V = 0$$

$$-\frac{h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2} = E\psi \rightarrow (3)$$

This is in the form Ĥψ = Eψ

This is an Eigen-value equation.

Let $\dfrac{8\pi^2 m}{h^2}E = k^2$ in eqn (2)

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of this equation is:

$$\psi = C \cos k\,x + D \sin k\,x \to (4)$$

at $x = 0 \to \psi = 0$

$$0 = C \cos 0 + D \sin 0$$

$$\therefore C = 0$$

Also $x = a \to \psi = 0$

$$0 = C \cos ka + D \sin ka$$

But $C = 0$

$$\therefore D \sin ka = 0 \qquad\qquad \longrightarrow (5)$$

$D \neq 0$ (because the wave concept vanishes)

i.e. $ka = n\pi$ where $n = 0, 1, 2, 3, 4\dots$ (quantum number)

$$k = \frac{n\pi}{a} \to (6)$$

Using this in eqn (4)

$$\psi_n = D \sin \frac{n\pi}{a} x \to (7)$$

Which gives permitted wave functions.

To find out the value of D, normalization of the wave function is to be done.

i.e. $\displaystyle\int_0^a \left|\psi_n^2\right| dx = 1 \to (8)$

using the values of ψ_n from eqn (7)

$$\int_0^a D^2 \sin^2 \frac{n\pi}{a} x \, dx = 1$$

$$D^2 \int_0^a \left[\frac{1 - \cos(2n\pi/a)x}{2} \right] dx = 1$$

$$\frac{D^2}{2} \left[\int_0^a dx - \int_0^a \cos \frac{2n\pi}{a} x \, dx \right] = 1$$

$$\frac{D^2}{2} \left[x - \frac{a}{2n\pi} \sin \frac{2n\pi}{a} x \right]_0^a = 1$$

$$\frac{D^2}{2} [a - 0] = 1$$

$$\frac{D^2}{2} a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

$$\because \sin^2 \theta = \left(\frac{1 - \cos 2\theta}{2} \right)$$

Hence the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \rightarrow (9)$$

Energy Eigen values:

From Eq. 6 &2

$$\frac{8\pi^2 m}{h^2} E = k^2 = \frac{n^2 \pi^2}{a^2}$$

Implies $\quad E = \frac{n^2 h^2}{8ma^2}$

Alternative Method (Operator method)

Energy Eigen values are obtained by operating the wave function 'ψ' by the energy operator (Hamiltonian operator).

$$\hat{H} = -\frac{h^2}{8\pi^2 m} \frac{d^2}{dx^2} + V$$

Inside the well 0 < X < a, V=0

$$\hat{H} = -\frac{h^2}{8\pi^2 m} \frac{d^2}{dx^2} \rightarrow (10)$$

The energy Eigen value eqn is

$$\hat{H}\psi = E\psi \rightarrow (11)$$

From equation (10) and (11)

$$-\frac{h^2}{8\pi^2 m}\frac{d^2\psi_n}{dx^2} = E\psi$$

i.e. $\quad -\frac{h^2}{8\pi^2 m}\frac{d^2\psi_n}{dx^2} = E\psi_n \rightarrow (12)$

It is the Eigen value equation.

Differentiating eqn (9)

$$\frac{d\psi_n}{dx} = \sqrt{\frac{2}{a}}\frac{n\pi}{a}\cos\frac{n\pi}{a}x$$

Differentiating again

$$\frac{d^2\psi_n}{dx^2} = -\sqrt{\frac{2}{a}}\left(\frac{n\pi}{a}\right)^2\sin\frac{n\pi}{a}x$$

$$\frac{d^2\psi_n}{dx^2} = -\left(\frac{n\pi}{a}\right)^2\psi_n$$

Using this eqn. in (12)

$$\frac{h^2}{8\pi^2 m}\left(\frac{n\pi}{a}\right)^2\psi_n = E\psi_n$$

$$E = \frac{n^2 h^2}{8ma^2} \rightarrow (13)$$

It gives the energy Eigen values of the particle in an infinite potential well.

n = 0 is not acceptable inside the well because $\psi_n = 0$. It means that the electron is not present inside the well which is not true. Thus the lowest energy value for *n* = 1 is called zero point energy value or ground state energy.

i.e. $E_{\text{zero-point}} = \dfrac{h^2}{8ma^2}$

The states for which n >1 are called exited states.

Wave functions, probability densities and energy levels for particle in an infinite potential well:

Let us consider the most probable location of the particle in the well and its energies for first three cases.

Case I → n=1

It is the ground state and the particle is normally present in this state.

The Eigen function is

$$\psi_1 = \sqrt{\frac{2}{a}} Sin \frac{\pi}{a} x \quad \because \text{from eqn (7)}$$

$\psi_1 = 0$ for $x = 0$ and $x = a$

But ψ_1 is maximum when $x = a/2$.
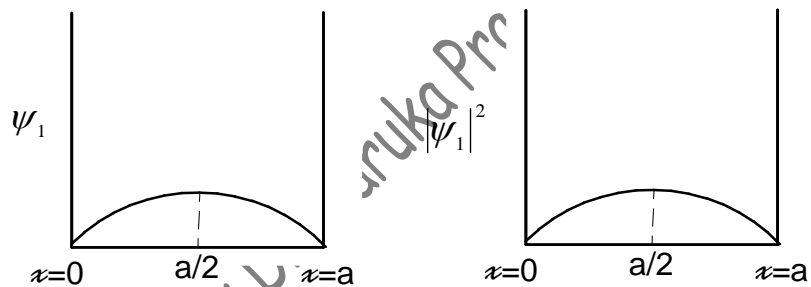
The plots of ψ_1 versus x and $|\psi_1|^2$ verses x are shown in the above figure.

$|\psi_1|^2 = 0$ for $x = 0$ and $x = a$ and it is maximum for $x = a/2$. i.e. in ground state the particle cannot be found at the walls, but the probability of finding it is maximum in the middle.

The energy of the particle at the ground state is

$$E_1 = \frac{h^2}{8ma^2} = E_0$$

Case II → n=2

In the first excited state the Eigen function of this state is

$$\psi_2 = \sqrt{\frac{2}{a}} Sin \frac{2\pi}{a} x$$

$\psi_2 = 0$ for the values $x = 0, a/2, a$.

Also ψ_2 is maximum for the values $x = a/4$ and $3a/4$.

These are represented in the graphs.

$|\psi_2|^2 = 0$ at $x = 0$, a/2, a, i.e. particle cannot be found either at the walls or at the centre. $\left|\psi_2\right|^2 =$ maximum $for\ x = \dfrac{a}{4}$, $x = \dfrac{3a}{4}$
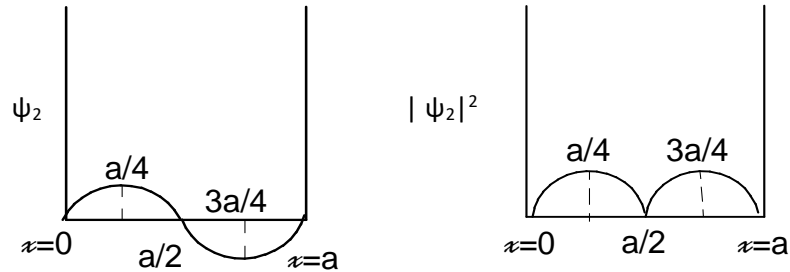
The energy of the particle in the first excited state is $E_2 = 4E_0$.

Case III → n=3

In the second excited state,

$$\psi_3 = \sqrt{\dfrac{2}{a}} Sin\dfrac{3\pi}{a}x$$

$\psi_3 = 0$, for X = 0, a/3, 2a/3 and a.

ψ_3 is maximum for x = a/6, a/2, 5a/6.

These are represented in the graphs.
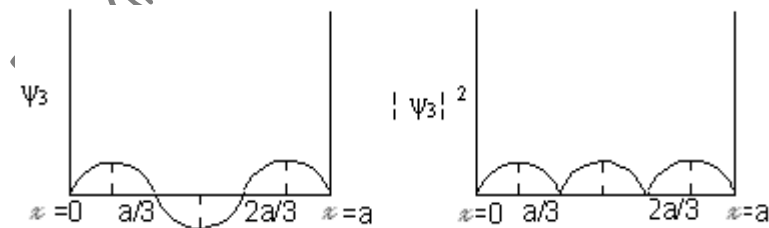
$|\psi_3|^2 = 0$ for $x = 0$, a/3, 2a/3 and a. $\left|\psi_3\right|^2 =$ maximum $for\ x = \dfrac{a}{6}$, $x = \dfrac{a}{2}$, $x = \dfrac{5a}{6}$

The energy of the particle in the second excited state is $E_3 = 9\ E_0$.

Energy Eigen values of a free particle:

A free particle is one which has zero potential. It is not under the influence of any force or field i.e. V = 0.

The Schrodinger equation is:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi = 0$$

or $\quad -\frac{h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2} = E\psi$

This equation holds good for free particle in free space in which V = 0.

With the knowledge of the particle in a box or a particle in an infinite potential well V = 0 holds good over a finite width 'a' and outside V = ∞. By taking the width to be infinite i.e. a = ∞, the case is extended to free particle in space. The energy Eigen values for a particle in an infinite potential well is

$$E = \frac{n^2 h^2}{8ma^2}$$

Where n =1, 2, 3, …

$$n = \frac{2a}{h}\sqrt{2mE}$$

Here when 'E' is constant, 'n' depends on 'a' as a→∞ *n*→∞. It means that free particle can have any energy. That is the energy Eigen values or possible energy values are infinite in number. It follows that energy values are continuous. It means that there is no discreteness or quantization of energy. Thus a free particle is a *'Classical entity'*.

Numericals:-

1. Calculate the number of photons emitted in 3 hrs. by a 60 watt sodium lamp. Given λ=589.3nm.

2. Light of wavelength 4047A^0 falls on a photoelectric cell with a sodium cathode. It is found that the photoelectric current ceases when a retarding potential of 1.02 volt is applied. Calculate the work function of the sodium cathode.

3. The most rapidly moving valence electron in metallic sodium at absolute zero temperature has a kinetic energy 3eV. Show that the de Broglie wavelength is $7A^0$.

4. Calculate the momentum of an electron possessing the de Broglie wavelength 6.62×10^{-11}m.

5. Find the phase and group velocities of an electron whose de-Broglie wavelength is 0.12nm.

6. Find the kinetic energy and group velocity of an electron with de-Broglie wavelength of 0.2nm (Jan 2011)

7. Calculate the wavelength of a 1kg object whose velocity is 1m/s and compare it with the wavelength of an electron accelerated by 100 volt ($\lambda_0 = 6.62 \times 10^{-34}$ m, $\lambda_e = 1.2 \times 10^{-10}$m and $\lambda_e = 1.8 \times 10^{23} \lambda_0$)

8. Calculate the de-Broglie wavelength associated with 400 gm cricket ball with a speed of 90Km/hr. (VTU August 2006)

9. Compare the energy of a photon with that of an electron when both are associated with wave length of 0.2nm. (VTU Feb 2006, August 2003, IAS- 1987)

10. Compare the energy of a photon with that of a neutron when both are associated with wave length of 0.1nm. Given the mass of neutron is 1.678×10^{-27}kg. (Jan 2009)

11. Calculate the wavelength associated with electrons whose speed is 0.01 of the speed of light. (VTU August 2004)

12. The velocity of an electron of a Hydrogen atom in the ground state is 2.19×10^6m/s. Calculate the wavelength of the de Broglie waves associated with its motion.

13. Compute the de Broglie wavelength for a neutron moving with one tenth part of the velocity of light (June-July 2011)

14. Estimate the potential difference through which a proton is needed to be accelerated so that its de Broglie wavelength becomes equal to $1A^0$, given that its mass is 1.673×10^{-27}kg.

15. Calculate the de Broglie wavelength associated with an electron with a kinetic energy of 2000eV. (VTU March 2006)

16. Evaluate de Broglie wavelength of Helium nucleus that is accelerated through 500V.

17. A particle of mass 0.511 MeV/c^2 has kinetic energy 100eV. Find its de-Broglie wavelength, where c is the velocity of light. (VTU Jan 2007, IAS 1978)

18. Compare the momentum, the total energy, and the kinetic energy of an electron with a de-Broglie wavelength of $1A^0$, with that of a photon with same wavelength.

19. Calculate the de Broglie wavelength of a proton whose kinetic energy is equal to the rest energy of the electron. Mass of proton is 1836 times of electron.

20. A fast moving neutron is found to have an associated de Broglie wavelength of 2×10^{-12}m. Find its kinetic energy, the phase and group velocities of the de Broglie waves ignoring the relativistic change in mass. Mass of neutron= 1.675×10^{-27}kg.

21. If an electron has a de Broglie wavelength of 3 nm, find its kinetic energy and group velocity, given that it has rest mass energy of 511 kev.(Dec 2010)

22. A Particle of mass 0.65 MeV/c^2 has kinetic energy 80eV. Find the de Broglie wavelength, group velocity and phase velocity of the de Broglie wave. (VTU Model Question Paper, Jan 2008, July 2008)

23. An electron has a wavelength of 1.66×10^{-10}m. Find the kinetic energy, phase velocity and group velocity of the de Broglie wave. (VTU August 1999)

24. Calculate the wavelength associated with an electron having kinetic energy 100eV. (VTU July 2002)

25. Calculate the wavelength associated with an electron raised through a potential difference of 2 kV. (VTU Feb 2002)

26. Find the energy of an electron moving in one dimension in an infinitely high potential box of width 1.0 A^0. (Ans:- 37.694 n^2 eV)

27. A spectral line of wavelength 546.1nm has a width 10-5nm. Estimate the minimum time spent by electrons in the excited state during transitions. (July 2007,Dec 2010)

28. An electron is bound in a one dimensional potential box which has a width 2.5 x 10-10m. Assuming the height of the box to be infinite, calculate the two lowest permitted energy values of the electron. (Ans:- 6.04eV and 24.16eV)

29. Calculate the lowest energy of a neutron confined to the nucleus, where nucleus is considered a box with a size of 10-14m. (Ans:- 6.15Mev)

30. Estimate the time spent by an atom in the excited state during the excitation and de-excitation processes, when a spectral line of wavelength 546 nm and width 10-14 is emitted (Jan 2011).

31. A particle is in motion along a line between $x=0$ and $x=a$ with zero potential energy. At points for which $x<0$ and $x>a$, the potential energy is infinite. The wave function for the particle in the nth state is given by

$$\Psi_n = D \sin\left(\frac{n\pi}{a}x\right)$$

Find the expression for the normalized wave function.

32. Show that $\phi(x)=e^{ikx}$ is acceptable eigen-function, where k is some finite constant. For a region $-a \le x \le a$; normalize the given eigen-function.

33. A particle is moving in one dimensional potential well of infinite height and of width 25A0. Calculate the probability of finding the particle in an interval of 5 A0 at a distances of a/2, a/3 and a , where a is the width of the well assuming that the particle is in its least state of energy.(Ans:- P1=0.3871, P2= 0.2937 and P3=0.0086)

34. A quantum particle confined to one-dimensional box of width 'a' is in its first excited state. What is the probability of finding the particle over an interval of (a/2) marked symmetrically at the centre of the box (Jan 2011).

Questions:-

1. State and explain the Heisenberg uncertainty principle. Using this principle, show that the electrons cannot reside in an atomic nucleus. (Jan 2007, Jan 2011)
2. State the exact statement of Heisenberg uncertainty principle. Name three pairs of physical variables for which this law holds true.
3. Derive time-independent Schrödinger wave equation. What is the physical significance of state function 'ψ' used in this equation? What conditions must it fulfill?
4. What is a wave function? Explain the properties of wave function? (July 2007,June 2011)
5. Write down the Schrödinger equation for a particle in one-dimensional box. Obtain the eigen functions and eigen values for this particle.
6. A particle is moving freely within one-dimensional potential box. Find out the eigen functions of the particle and show that it has discrete eigen values.
7. Find the expression for the energy state of a particle in one-dimensional box.
8. What do you mean by an operator? Write the operators associated with energy and momentum.
9. Set up time-independent one-dimensional Schrödinger equation.
10. What is normalization of a wave function? What are the physical significance and properties of wave function?

11. What are eigen values and eigen functions? Discuss the nature of eigen values and eigen functions.

12. Give the Max Born's interpretation of wave function and explain the normalization conditions.

13. Solve the Schrödinger wave equation for the allowed energy values in the case of particle in a box. (June 2011)

14. Using Schrödinger wave equation for a particle in one-dimensional well of infinite height, discuss about energy eigen values.

15. Describe zero-point energy.

16. Assuming the time independent Schrödinger wave equation, discuss the solution for a particle in one-dimensional potential well of infinite height. Hence obtain the normalized wave function (Jan 2008)

17. What is the physical interpretation of wave function? How a free particle wave function signifies a particle in space and momentum?

18. Solve the Schrödinger wave equation for the one-dimensional potential well defined by

$$V(x) = \infty \qquad for \ x < 0 \ and \ x > a$$
$$V(x) = 0 \qquad for \ 0 \le x \le a$$

19. Find the eigen value and eigen functions for an electron in one dimensional potential well of infinite height (July 2007,Jan 2011)

20. Discuss the wave functions, probability densities and energy levels for a particle in a box. (Jan 2007)

B. M. S. Institute of Technology and Management
[Approved by AICTE NEW DELHI, Affiliated to VTU BELGAUM]
DEPARTMENT OF PHYSICS

COURSE MATERIAL   SUBJECT: - Engineering Physics
MODULE-IV   SUBJECT CODE: - 18 PHY 12 /22

LASER

The word Laser stands for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation. It is a device which amplifies light. It has properties like Coherence, Unidirectional, Monochromatic, Focus ability, etc.

Interaction of an electromagnetic wave with matter requires certain conditions. The interaction leads to transition of an atom or a molecule from one energy state to another. If the transition is from lower state to higher state it absorbs the incident energy. If the transition is from higher state to lower state it emits a part of its energy.

If ΔE is the difference between the two energy levels,

Then $\Delta E = (E_2 - E_1)$ Joule

According to Max Planck, $\Delta E = h\nu = (E_2 - E_1)$

     $\nu = (E_2 - E_1)/h$ Hz.

Three types of interactions, which are possible:

1) Induced absorption/ Stimulated absorption/absorption
2) Spontaneous emission
3) Stimulated emission.

Emission or Absorption takes through quantum of energy called photons. $h\nu$ is called quantum energy or photon energy.

$h = 6.626 \times 10^{-34}$ Joules is Planck's constant and 'ν' is the frequency.

1) Induced absorption/ Stimulated absorption/absorption:
Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state, wherein the difference in energy of the two states is the energy of the photon.

Consider the system having two energy states E_1 and E_2, $E_2 > E_1$. When a photon of energy $h\nu$ is incident on an atom at level E_1, the atom goes to a higher energy level by absorbing the energy.
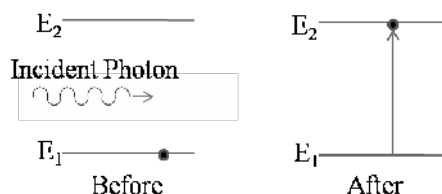
When an atom is at ground level (E_1), if an electromagnetic wave of suitable frequency ν is applied to the atom, there is possibility of getting excited to higher level (E_2). The incident photon is absorbed. It is represented as

Atom + Photon → Atom*

The frequency of the absorbed photon is

$\nu = (E_2-E_1)/h$.

The rate of absorption is proportional to $N_1 U_\nu$

Where 'N_1' is the number density of the lower energy state, 'U_ν' is the energy density of incident radiation.

Rate of absorption = $B_{12}N_1U_\nu$

Where 'B_{12}' is the proportionality constant called Einstein Coefficient of induced absorption.

1. *Spontaneous Emission:*

The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission. Let 'E_1' and 'E_2' be two energy levels in a material, such that $E_2>E_1$. E_1 is ground level and E_2 is the higher level. $h\nu=E_2-E_1$ is the difference in the energy. The atom at higher level (E_2) is more unstable as compared to that at lower level (E_1).

The time taken by the atom to remain in the excited state is normally very short (order of 10^{-8} s) and it is called life time of the atom. In spontaneous emission atom emits the photon without the aid of any external energy. It is called spontaneous emission. The process is represented as

Atom* → Atom + Photon

The photons emitted in spontaneous emission may not have same direction and phase similarities. It is incoherent.

Ex: Glowing electric bulbs, Candle flame etc.
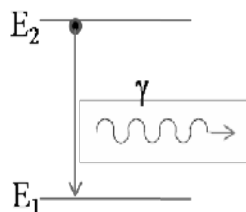
Spontaneous emission depends on N_2 which is the number of atoms present in the higher level.

The rate of spontaneous emission = $A_{21}N_2$

Where 'A_{21}' is the proportionality constant called Einstein coefficient of spontaneous emission.

2. *Stimulated Emission:*

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of just the right energy due to which the system transits from a higher energy state to a lower energy state.

$$E_2 - E_1 = \Delta E = h\nu$$
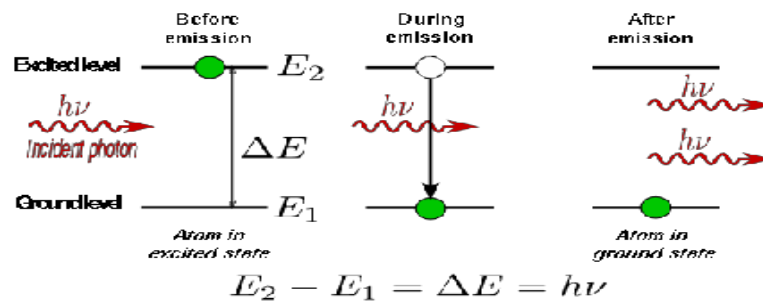
The photon thus emitted is called stimulated photon and will have the same phase, energy and direction of movement as that of the passing photon called the stimulation photon.

Initially the atom is at higher level E_2. The incident photon of energy $h\nu$ forces the atom to get de-excited from higher level E_2 to lower level E_1. i.e. $h\nu = E_2 - E_1$ is the change in energy.

The incident photon stimulates the excited atom to emit a photon of exactly the same energy as that of the incident photons. The emitted two photons have same phase, frequency, direction and polarization with the incident photon. This kind of action is responsible for lasing action.

Atom* + Photon → Atom + (Photon + Photon)

The rate of stimulated emission is directly proportional to $N_2 U_\nu$, where 'N_2' is the number of atoms present in the higher energy level and 'U_ν' is the energy density.

The rate of stimulated emission = $B_{21} N_2 U_\nu$, where 'B_{21}' is the proportionality constant called Einstein's Coefficient of stimulated emission.

Einstein's A & B Coefficients:-

(*Note: - First explain the phenomena of spontaneous emission, stimulated emission and spontaneous absorption and continue as explained below*)

At thermal equilibrium,

Rate of absorption = (Rate of spontaneous emission + Rate of stimulated emission)

$$B_{12} N_1 U_\nu = A_{21} N_2 + B_{21} N_2 U_\nu$$

$$U_\nu (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$U_\nu = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

i.e $\quad U_\nu = \dfrac{A_{21}}{B_{21}} \left[\dfrac{N_2}{\left(\dfrac{B_{12}}{B_{21}} N_1 - N_2 \right)} \right]$

$$= \dfrac{A_{21}}{B_{21}} \left[\dfrac{1}{\left(\dfrac{B_{12} N_1}{B_{21} N_2} \right) - 1} \right] \qquad \rightarrow (1)$$

By Boltzmann's law,

$$N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{KT} \right)} = N_1\ e^{-h\nu/KT}$$

i.e., $\quad N_1/N_2 = e^{h\nu/KT}$

Eqn. (1) becomes

$$U_\nu = \dfrac{A_{21}}{B_{21}} \left[\dfrac{1}{\left(\dfrac{B_{12}}{B_{21}} e^{\left(\frac{h\nu}{kT} \right)} - 1 \right)} \right] \qquad \rightarrow (2)$$

By Planck's law,

$$U_\nu = \dfrac{8\pi h \nu^3}{c^3} \left[\dfrac{1}{\left(e^{\left(\frac{h\nu}{kT} \right)} - 1 \right)} \right] \qquad \rightarrow (3)$$

Comparing equation (2) & (3)

$$\dfrac{A_{21}}{B_{21}} = 8\pi h\nu^3/c^3 \quad \& \quad \dfrac{B_{12}}{B_{21}} = 1 \quad \text{i.e. } B_{12} = B_{21}$$

The probability of induced adsorption is equal to the stimulated emission.

Conclusions of Einstein co-efficient:

Dependence of nature of emission on frequency:

$$\text{Consider } \dfrac{A_{21}}{B_{21}} = 8\pi h\nu^3/c^3 \qquad \rightarrow (1)$$

If A_{21} has high value, the probability of spontaneous emission is high. If B_{21} has high value, the probability of stimulated emission is high.

Further $\quad \dfrac{A_{21}}{B_{21}} \ \alpha \ \nu^3$

Since $\nu = \Delta E / h$, in normal condition, the energy difference between the two levels E_1 and E_2 is large

$$\frac{A_{21}}{B_{21}} >> 1 \quad \text{or} \quad A_{21} >> B_{21}$$

Thus the probability of spontaneous emission is more than the stimulated emission.

System in thermal equilibrium:

According to Planck's law

$$U_\nu = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{\frac{h\nu}{KT}}-1}\right) \qquad \rightarrow (2)$$

Using eqs (1) & (2) and rearranging, we have

$$\frac{A_{21}}{B_{21}U_\nu} = e^{h\nu/kT} -1 \qquad \rightarrow (3)$$

Case-1: $h\nu >> kT$

When the frequency of radiation is high $h\nu >> kT$ i.e. $e^{h\nu/kT} >> 1$

Hence in eqn (3) $\dfrac{A_{21}}{B_{21}} >> 1$ i.e. $A_{21} >> B_{21}$

That is spontaneous emission is more than the stimulated emission.

Case-2: $h\nu \approx kT$

For $h\nu \approx kT$, $e^{h\nu/kT}$ will be low and comparable to 1

Therefore A_{21} and B_{21} become comparable, i.e. stimulated emission became significant.

Case-3: $h\nu << kT$

For $h\nu << kT$, $(e^{h\nu/kT}-1) << 1$ and $\dfrac{A_{21}}{B_{21}} << 1$ or $B_{21} >> A_{21}$.

That is stimulated emission is more for lower frequency.

For microwaves frequency is very less, so achieving $B_{21} >> A_{21}$ is easy with microwaves. Therefore first MASER (Microwave Amplification by Stimulated Emission of Radiation) came to exist.

Non-equilibrium conditions leading to amplification:

$$\text{We have } \frac{\text{Rate of emission}}{\text{Rate of absorption}} = \frac{A_{21}N_2 + B_{21}N_2U_\gamma}{B_{12}N_1U_\gamma} = \frac{N_2}{N_1}\left[\frac{A_{21}+B_{21}U_\gamma}{B_{12}U_\gamma}\right]$$

According to Einstein's theory we have $B_{12} = B_{21}$

$$\frac{\text{Rate of emission}}{\text{Rate of absorption}} = \frac{N_2}{N_1}\left[\frac{A_{21}}{B_{21}U_\gamma} + 1\right] \qquad \rightarrow (4)$$

From eqn (3) if $\Delta E \ll KT$ i.e. $h\nu \ll KT$

Then $A_{21}/(B_{21}U_\nu) \ll 1$

Hence eqn (4) can be written as

$$\frac{\text{Rate of emission}}{\text{Rate of absorption}} = \frac{N_2}{N_1}$$

Under normal conditions N_2 is always less than N_1

1. **_Meta Stable State:_** It is the state where the atoms get excited and remains in the excited state for longer time than the normal state.
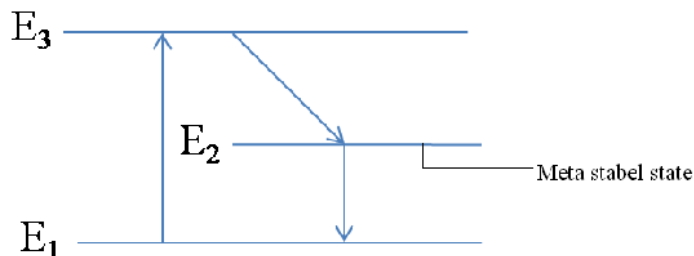
This state plays an important role in lasing action. In metastable state, atoms stay of the order of 10^{-3} to 10^{-2} second. In excited state other than metastable atom stay of order of 10^{-8} second.

2. **_Population Inversion:_** It is the state of the system at which the population of a higher energy level is greater than that of the lower energy level.

Let E_1, E_2, E_3 be the energy levels of the system $E_3 > E_2 > E_1$. E_2 is the metastable state of the system. Atoms get excited from the state E_1 to E_3 by means of external source and stay there for short time. These atoms undergo spontaneous transitions to E_2 and E_1. The atoms at the state E_2 stay for longer time. A stage is reached in which the number of atoms at state E_2 is more than the number of atoms at E_1 which is known as population inversion.

3. **_Pumping:_** The process of producing population inversion is called pumping. It is the process of exciting atoms from lower energy level to higher energy level. It can be achieved by different methods.

   a. **_Optical pumping:_** Using high intensity light or by operating flash tube. Ex:Ruby Laser.
   b. **_Electric Discharge:_** By applying very high potential between the plates of discharge tube gas gets discharge leads to pumping. Ex: Argon Laser.
   c. **_Atom-Atom Collision:_** Excited atoms collide with other types of atom and transfer its energy to bring other atoms to excited state. Ex: He-Ne Laser.
   d. **_Chemical Method:_** Exothermic chemical reactions liberate energy. This liberated energy is used in pumping the atoms. Ex: Dye Laser.
   e. **_Using Current :_** In semiconductor diode laser the tuning of current input brings the charge carriers to achieve population inversion.

4. **_Requisites of a Laser System:_**
   1) The excitation source for pumping action.

2) Active medium for population inversion.
3) Laser cavity, an active medium bounded by two mirrors. (Resonator/ Fabry-Perot resonator)

CO₂ LASER: Construction and Working (Ref: http://www.daenotes.com)

CO₂ Laser (The molecular gas laser)

The CO_2 stands for carbon dioxide. In CO_2 laser the laser light takes place within the molecules of carbon dioxide rather than within the atoms of a pure gas. Therefore CO_2 gas laser is considered the type of molecular gas laser. Importantly note that CO_2 lasers use carbon dioxide as well as Helium & Nitrogen as its active medium. In a molecular gas laser, laser action is achieved by transitions between vibrational and rotational levels of molecules. Its construction is simple and the output of this laser is continuous. In CO_2 molecular gas laser, transition takes place between the vibrational states of Carbon dioxide molecules.

CO₂ Molecular gas laser

It was the first molecular gas laser developed by Indian born American scientist Prof.C.K.N.Pillai. It is a four level laser and it operates at 10.6 µm in the far IR region. It is a very efficient laser.

Energy states of CO₂ molecules.

A carbon dioxide molecule has a carbon atom at the center with two oxygen atoms attached, one at both sides. Such a molecule exhibits three independent modes of vibrations. They are

      a)   Symmetric stretching mode.
      b)   Bending mode
      c)   Asymmetric stretching mode.

a. Symmetric stretching mode

In this mode of vibration, carbon atoms are at rest and both oxygen atoms vibrate simultaneously along the axis of the molecule departing or approaching the fixed carbon atoms.
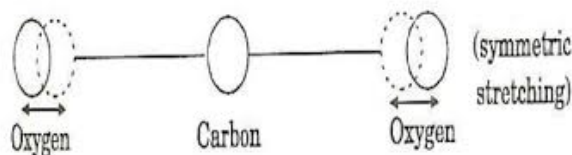
b. Bending mode:

In this mode of vibration, oxygen atoms and carbon atoms vibrate perpendicular to molecular axis.
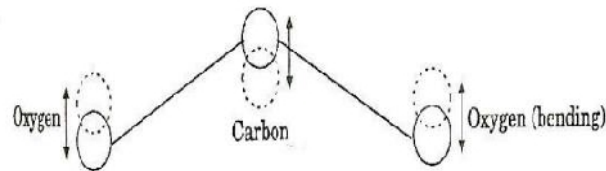
c. **Asymmetric stretching mode**:

In this mode of vibration, oxygen atoms and carbon atoms vibrate asymmetrically, i.e., oxygen atoms move in one direction while carbon atoms in the other direction.
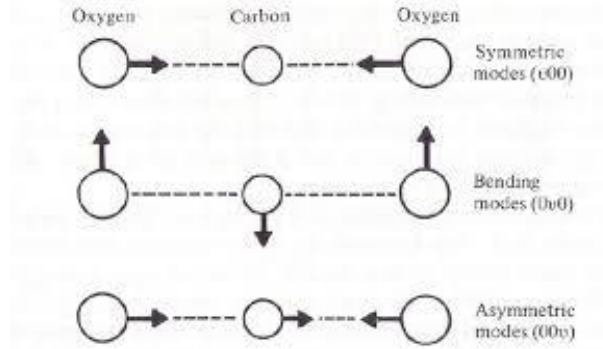
Principle:

The active medium is a gas mixture of CO_2, N_2 and He in the ratio of 1:2:3. The laser transition takes place between the vibrational states of CO_2 molecules.

Construction:

It consists of a quartz tube 5 m long and 2.5 cm in the diameter. This discharge tube is filled with gaseous mixture of CO_2 (active medium), helium and nitrogen with suitable partial pressures.

The terminals of the discharge tubes are connected to a D.C power supply. The ends of the discharge tube are fitted with NaCl Brewster windows so that the laser light generated will be polarized.

Two concave mirrors one fully reflecting and the other partially form an optical resonator.
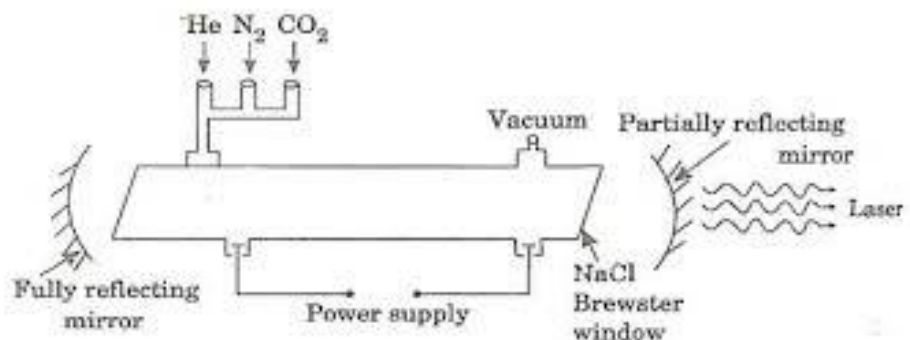
Working:

Figure shows energy levels of nitrogen and carbon dioxide molecules.
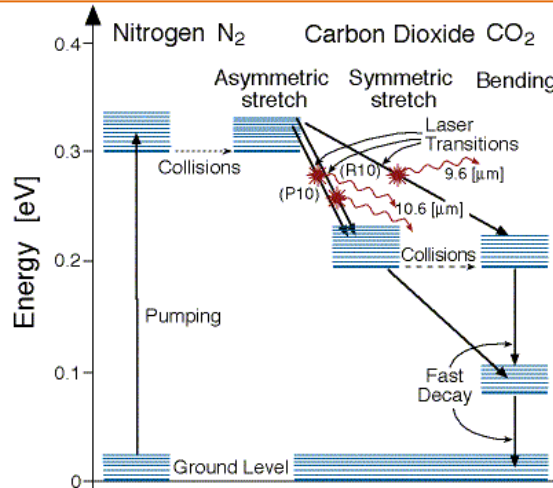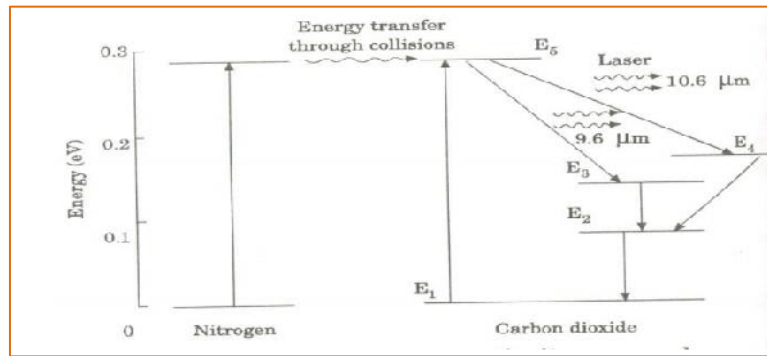
When an electric discharge occurs in the gas, the electrons collide with nitrogen molecules and they are raised to excited states. This process is represented by the equation

$$N_2 + e^* = N_2^* + e$$
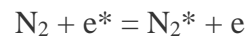
N_2 = Nitrogen molecule in ground state e^* = electron with kinetic energy
N_2^* = nitrogen molecule in excited state e= same electron with lesser energy

Now N_2 molecules in the excited state collide with CO_2 atoms in ground state and excite to higher electronic, vibrational and rotational levels.
This process is represented by the equation $N_2^* + CO_2 = CO_2^* + N_2$

N_2^* = Nitrogen molecule in excited state. CO_2 = Carbon dioxide atoms in ground state
CO_2^* = Carbon dioxide atoms in excited state N_2 = Nitrogen molecule in ground state.

Since the excited level of nitrogen is very close to the E_5 level of CO_2 atom, population in E_5 level increases.

As soon as population inversion is reached, any of the spontaneously emitted photon will trigger laser action in the tube. There are two types of laser transition possible.

1.Transition E₅ to E₄ :
This will produce a laser beam of wavelength 10.6μm
2.Transition E₅ to E₃
This transition will produce a laser beam of wavelength 9.6μm. Normally 10.6μm transition is more intense than 9.6μm transition. The power output from this laser is 10kW.

1. Type: It is a molecular gas laser.
2. Active medium: A mixture of CO_2 , N_2 and helium or water vapour is used as active medium
3. Pumping method: Electrical discharge method is used for Pumping action
4. Optical resonator: Two concave mirrors form a resonant cavity
5. Power output: The power output from this laser is about 10kW.
6. Nature of output: The nature of output may be continuous wave or pulsed wave.
7. Wavelength of output: The wavelength of output is 0.6 μm and 10.6 μm.

Advantages:
1. The construction of CO_2 laser is simple
2. The output of this laser is continuous.
3. It has high efficiency
4. It has very high output power.
5. The output power can be increased by extending the length of the gas tube.

Disadvantages:
1. The contamination of oxygen by carbon monoxide will have some effect on laser action
2. The operating temperature plays an important role in determining the output power of laser.
3. The corrosion may occur at the reflecting plates.
4. Accidental exposure may damage our eyes, since it is invisible (infra-red region) to our eyes.

Applications:
1. High power CO_2 laser finds applications in material processing, welding, drilling, cutting soldering etc.
2. The low atmospheric attenuation (10.6μm makes CO_2 laser suitable for open air communication.
3. It is used for remote sensing
4. It is used for treatment of liver and lung diseases.
5. It is mostly used in neuro surgery and general surgery.
6. It is used to perform microsurgery and bloodless operations.

Gallium-Arsenide Laser: Semiconductor laser:

A Semiconductor diode laser is a specially fabricated p-n junction device that emits coherent light when it is forward biased. In the case of germanium and silicon based diodes, this energy is released in the form of heat because of recombination of carriers take place through interaction with the atoms of the crystal. But in the case of GaAs, the energy is released in the form of photons as the atoms of the crystal are not involved in the release of energy. The wavelength of the emitted photon depends upon the activation energy of the crystal.

Construction: A schematic diagram of semiconductor laser is as shown in the figure. The diode is very small size with sides of the order of 1mm. The junction lies in a horizontal plane. The top and bottom surfaces are metalized and Ohmic contacts are provided for external connection. The front and rear faces are polished. The polished faces constitute the Fabry-perot resonator. The other two faces are roughened to

prevent lasing action in that direction. The active region consists of about 1μm thickness.

The emitted photon stimulates the recombination of the other carriers.
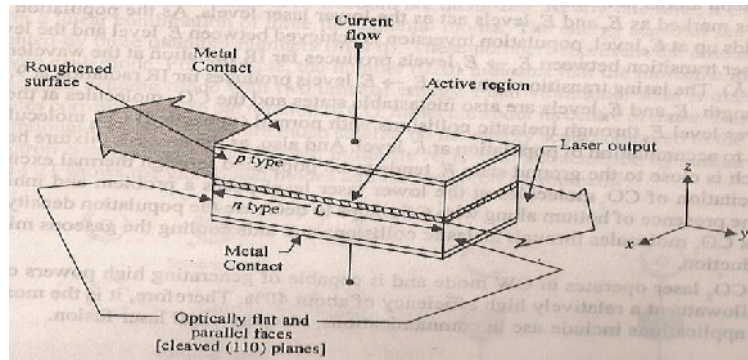
Cross sectional view of p-n junction diode laser.

Working: The energy band diagram of heavily doped pn-junction is as shown unbiased condition. At thermal equilibrium, the Fermi level is uniform across the junction. Because of very high doping on n-side, Fermi level is pushed into the conduction band and electrons occupy the portion of the conduction band lying below the Fermi level. On P-side, the Fermi level lies within the valence band and holes occupy the portion of the valence band that lies above the Fermi level. When the junction is forward biased electrons and holes are injected into the junction region in high concentrations. At low forward current, the electron-holes recombination results in spontaneous emission of photons and the junction acts as a LED. As the forward current is increased gradually and when it reaches a threshold value the carrier concentration in the junction region there will be large concentrations of electrons within the band. As a result condition of population inversion is attained in the narrow region. This narrow zone in which polulation inversion occurs is called as an active region, at that stage a photon emitted spontaneously triggers stimulated emission. This stimulated electron-hole recombination produces coherent radiation.
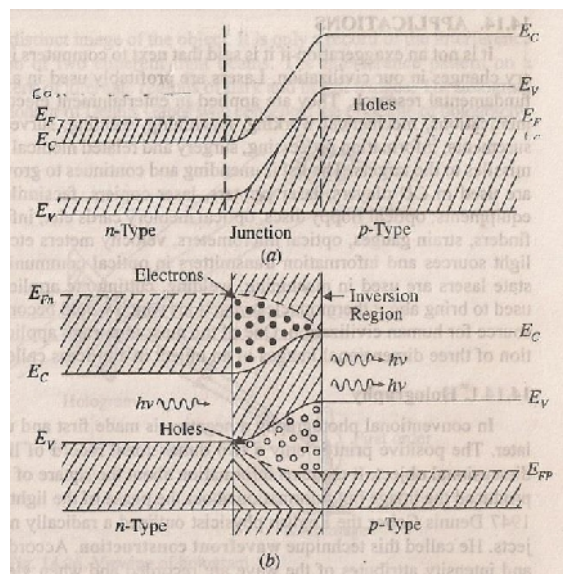
Energy level diagram of p-n junction diode laser
(a) Before biasing   (b) After biasing.

The stimulated electron-hole recombination causes emission of coherent radiation of very narrow bandwidth. At room temperature, GaAs laser emits light of wavelength $9000A^0$ and GaAsP laser radiates at $6500A^0$.

Advantages of semiconductor laser:
1. They are compact
2. They are efficient
3. They are highly stable

Applications of Laser

Laser Range Finder

    To knock down an enemy tank, it is necessary to range it very accurately. Because of its high intensity and very low divergence even after travelling quite a few kilometres, laser is ideally suited for this purpose. The laser range finders using neodymium and carbon dioxide lasers have become a standard item for artillery and tanks. These laser range finders are light weight and have higher reliability and superior range accuracy as compared to the conventional range finders. The laser range finder works on the principle of a radar. It makes use of the characteristic properties of the laser beam, namely, monochromaticity, high intensity, coherency, and directionality. A collimated pulse of the laser beam is directed towards a target and the reflected light from the target is received by an optical system and detected. The time taken by the laser beam for the to and fro travel from the transmitter to the target is measured. When half of the time thus recorded is multiplied by the velocity of light, the product gives the range, i.e., the distance of the target. The laser range finder is superior to microwave radar as the former provides better collimation or directivity which makes high angular resolution possible. Also, it has the advantage of greater radiant brightness and the fact that this brightness is highly directional even after travelling long distances, the size of the emitting system is greatly reduced. The high monochromaticity permits the use of optical band pass filter in the receiver circuit to discriminate between the signal and the stray light noise.

    A typical laser range finder can be functionally divided into four parts: (i) transmitter, (ii) receiver, (iii) display and readout, and (iv) sighting telescope. An earlier version of a laser range finder is schematically shown in the figure as showed below:-
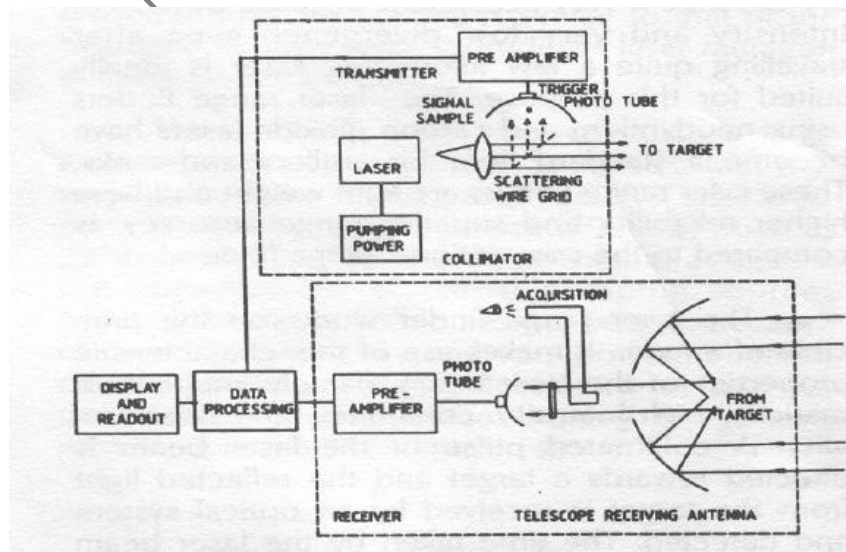
The transmitter uses a Q-switched Nd:YAG laser which sends out single, collimated and short pulse of laser radiation to the target. A scattering wire grid directs a small sample of light from

the transmitter pulse on to the photodetector, which after amplification is fed to the counter. This sample of light starts the counter. The reflected pulse, received by the telescope, is passed through an interference filter to eliminate any extraneous radiation. It is then focused on to another photodetector. The resulting signal is then fed to the counter. A digital system converts the time interval into distance. The range, thus determined by the counter, is displayed in the readout. The lighting telescope permits the operator to read the range while looking at the target Special circuits have been used to eliminate Spurious signals with the help of range gating and to make the use of laser range finder Possible under all weather conditions for which the targets can be seen visually through the sighting telescope. The modern versions of the laser range finders Use either high repetition pulsed Nd:YAG laser or carbon dioxide laser with range gating system. In ranging a target about 10 km away using these systems, accuracy within 5 m is easily obtained. The laser range finders of medium range (up to 10 km) are used in several Defense areas.

Engineering (Data storage)

Almost from the invention of the **laser**, researchers were considering the possibilities of optical data storage. Throughout the 1960s and 1970s, a number of companies were at work on optical data storage systems, held back in large part by the cost and performance level of available lasers. In 1982, Sony Corp. revolutionized the music industry with the introduction of the **compact disc** (CD). CD-ROM systems for computers quickly followed, expanding the capability of desktop computing. More recently, writable optical disks have been developed, and considerable ground has been gained in holographic data storage technology.

Optical data storage refers to any method of storing data using **light**. The most common method is optical disk, which offers a data **density** considerably higher than magnetic methods. There are three types of optical disks: ROM, or read only memory; WORM, or write once, read many times; and MO, or magneto optical disk, a disk which, like magnetic computer disks, can be repeatedly written on and repeatedly erased.

For ROM systems, information is recorded on a master disk by pulsed laser. The laser beam is varied or modulated, such that digital data is encoded in the pulses. The beam heats up and distorts a thermally sensitive layer on the master disk, recording a bit of data as a depression in the surface. The depressions are submicron in size, separated by grooves spaced 1.6 microns apart.

Once the master disk is created, copies can be produced quickly and cheaply (the cost of a CD is estimated at less than $1). Injection molded polycarbonate replicas made from the master disk are coated with **aluminum** to increase reflectivity, then sealed in protective plastic. The data retrieval system consists of a low power (3 mW), continuous wave **diode** laser, a series of **optics** to focus and circularize the beam before it reaches the CD, more optics to check that the beam is reading the proper area of CD at the correct location, and a detector to decode the signal. The disk spins, and the read head containing the laser and optics scans across it. The beam is reflected from the depressions on the optical disk, and the detector reads variations in the intensity and polarization of the light. These variations are decoded and converted to an electrical signal. In the case of a music CD, the electrical signal is transmitted to a speaker, the speaker diaphragm vibrates, and the result is music. CDs are capable of carrying prodigious amounts of information, over 600 megabytes on a single disk. In addition to the music and film formats, CD-ROMs bring extensive databases to the desktop computer, and the average user's fingertips. World atlases, encyclopedias, and comprehensive periodical indexes are just a few of the CD-ROM products available.

Write once, read many, or WORM systems, are a bit more complicated than ROM systems. Though they have essentially the same optical system for data retrieval, for writing operations they require a more powerful laser and a modified storage disk. Writable WORM disks are made of different material than consumer CD-ROMs. Typically, a thermally sensitive film is sandwiched between layers of **glass** or plastic. During the write phase, digital data is converted into an optical signal by varying or modulating a laser beam. The laser puts out about 30 mW of power, since it has to be capable of distorting the write layer. The tightly focused, modulated beam shines through the transparent glass or plastic and hits the thermally sensitive layer, heating it to create distortions that represent bits of data. These distortions are usually either bumps, depressions, or variations in opacity in the material that will make changes in the reflectivity of the surface. To read the disk, the laser/read assembly is scanned over the surface at lower power, and a detector reads and decodes variations in the surface reflectivity to obtain the original signal. Once recorded, the data cannot be rewritten, and short of destruction of the disk, cannot be erased. WORM disks are being used for archival purposes or in documentation-intensive applications such as insurance, banking, or government.

Other methods of optical data storage are being explored, particularly holographic data storage. A hologram is simply an image recorded using optical phase information that makes it appear three-dimensional. A pattern of 1s and 0s can be recorded as easily as a picture, and more quickly than the corresponding number of 1s and 0s can be sequentially stored. Though groups have demonstrated the feasibility of this approach, the development of a rewritable material capable of recording holograms and offering long term storage stability is still in its early stages. Significant electronic development is required as well. For the time being, optical disk technology seems to be securely in the forefront.

Despite minor drawbacks, optical data techniques are the technology of the future for data storage. The potential for great strides forward in performance clearly exists.

Properties of laser:

1. *Coherence*: The emitted radiation after getting triggered is in phase with the incident radiation.
   Coherence is of two types
   a. Temporal or time coherence: In a source like sodium lamp, two waves of slightly different wavelengths are given out. These waves have slightly different coherence time (Δt). A definite phase relationship exists between the two types of waves. This is known as coherence of the beam. The coherence length L is determined by the relation
   $$L = c.\Delta t$$
   b. Spatial Coherence: A laser beam is said to possess spatial coherence if the phase difference of the waves crossing the two points on a plane perpendicular to the direction of propagation of the beam is time independent. Spatial Coherence is also termed as transverse or lateral coherence.

2. *Monochromaticity*: The laser beam is highly monochromatic than any other radiations.

3. *Unidirectionality*: Laser beam travels in only one direction. It can travel long distance without spreading.
4. *Focusability*: A laser beam can be focused to an extremely fine spot.
5. *Intensity*: The power output of the laser may vary from few milliwatts to few kilowatts. But this energy is concentrated in a beam of very small cross section. The intensity of laser beam is approximately given by

$$I = \left(\frac{10}{\lambda}\right)^2 P \quad Wm^{-2}$$

Where p is the power radiated by laser.
In case of He-Ne laser, $\lambda = 6328 \times 10^{-10}$m and P= 10x 10^{-3}W,
the corresponding intensity is

$$I = \left(\frac{1}{6328 \times 10^{-10}}\right)^2 100 \times 10^{-3} \quad = 2.5 \times 10^{11} \quad Wm^{-2}$$

To obtain the above intensity from tungsten bulb, the temperature would have to be raised to 4.6×10^6K. The normal operating temperature of the bulb is approximately 2000K.

Prepared by Dr. Daruka Prasad B

B. M. S. Institute of Technology and Management
[Approved by AICTE NEW DELHI, Affiliated to VTU BELGAUM]
DEPARTMENT OF PHYSICS

COURSE MATERIAL   SUBJECT: - Engineering Physics
MODULE-V   SUBJECT CODE: - 18 PHY 12 /22

Syllabus

Material science

Quantum Free electron theory of metals: Review of classical free electron theory, mention of failures. Assumptions of Quantum Free electron theory, Mention of expression for density of states, Fermi-Dirac statistics (qualitative), Fermi factor, Fermi level, Derivation of the expression for Fermi energy, Success of QFET.

Physics of Semiconductor: Fermi level in intrinsic semiconductors, Expression for concentration of electrons in conduction band, Hole concentration in valance band (only mention the expression), Conductivity of semiconductors(derivation), Hall effect, Expression for Hall coefficient(derivation)

Dielectric materials: polar and non-polar dielectrics, internal fields in a solid, Clausius-Mossotti equation(Derivation), mention of solid, liquid and gaseous dielectrics with one example each. Application of dielectrics in transformers.

Numerical problems

Quantum Free electron theory of metals:

Review of Classical free electron theory and mention their failures:-

1. Give the assumptions of the classical free electron theory.

The main assumptions of classical free electron theory are:

A metal is imagined as the structure of 3-dimensional array of ions in between which, there are free moving valence electrons confined to the body of the material. Such freely moving electrons cause electrical conduction under an applied field and hence referred to as conduction electrons. The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T is given by 3/2 KT, where K is a Boltzmann constant. It is related to the kinetic energy.

$$3/2 \text{ KT} = ½ \text{ mv}_{th}^2$$

Where v_{th} is the thermal velocity same as root mean square velocity.

The electric potential due to the ionic cores is taken to be essentially constant throughout the body of the metal and the effect of repulsion between the electrons is considered insignificant. The electric current in a metal due to an applied field is a consequence of the drift velocity in a direction opposite to the direction of the field.

2. Explain the terms: a) Drift velocity b) Relaxation time c) Mean free path d) Mean collision time for free electrons.

a) *Drift velocity(v_d)*: The velocity of electrons in the steady state in an applied electric field is called drift velocity.

b) *Relaxation time (τ_r)*: From the instant of sudden disappearance of an electric field across a metal, the average velocity of the conduction electrons decays exponentially to zero, and the time required in this process for the average velocity to reduce to (1/e) times its value is known as Relaxation time.

c) *Mean free path (λ)*: The average distance travelled by the conduction electrons between successive collisions with lattice ions.

d) *Mean collision time (τ)*: The average time that elapses between two consecutive collisions of an electron with the lattice points is called mean collision time.

$$\tau = \lambda/v$$

where 'λ' is the mean free path, $v \approx v_{th}$ is velocity same as combined effect of thermal & drift velocities.

3. Describe under what circumstances, the relaxation time is equal to the mean collision time.

The relaxation time τ_r and mean collision time τ are related as

$$\tau_r = \frac{\tau}{1-<cos\theta>}$$

For isotropic scattering or symmetrical scattering

$<cos\theta> = 0$ Then $\tau_r = \tau$

4. Derive the expression for drift velocity.
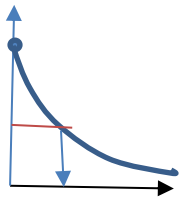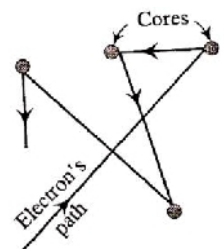
Consider a conductor subjected to an electric field. In the steady state, conduction electrons move with constant velocity. If 'm' is the mass of the electron, 'v' is the velocity same as the drift velocity 'v_d', 'τ' is the mean collision time, then the resistance force 'F_r' offered to its motion is given by

$$F_r = \frac{m\,v_d}{\tau}$$

If 'e' is the charge on an electron, 'E' is the electric field, the driving force on the electron is

$$F = eE$$

CURRENT CARRYING CONDUCTOR

In the steady state $F = F_r$

i.e. $\dfrac{m\,v_d}{\tau} = eE$

The drift velocity $v_d = \dfrac{eE}{m}\tau$
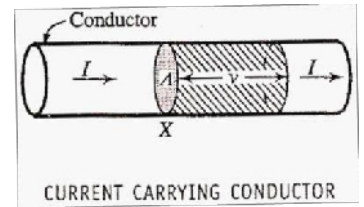
5. Using the free electron model derive an expression for electrical conductivity in metals.

Consider the motion of an electron in a conductor in an influence of an electric field. If e is the charge, m is the mass of an electron, E is the electric field. The force on an electron is

$$F = eE$$

Also from Newton's laws of motion $\qquad F = m\dfrac{dv}{dt}$

$\therefore \quad eE = m\dfrac{dv}{dt}$

or $\qquad dv = \dfrac{eE}{m}dt$

Integrating both sides

$\int dv = \int \dfrac{eE}{m}dt$

$v = \dfrac{eE}{m}t$

If the time of traverse is taken to be collision time 'τ' and 'v' is taken as average velocity $\bar{v}$

LATTICE SCATTERING

$\bar{v} = \dfrac{eE}{m}\tau$

We have $\sigma = \dfrac{J}{E}$; where 'J' is the current density.

But $J = \dfrac{I}{A}$

Where 'I' is the current and 'A' is the area of cross section of the conductor
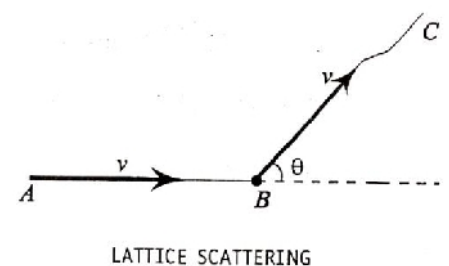
Therefore $\sigma = \dfrac{I}{AE}$

The distance travelled in unit time is $\bar{v}$

Volume sweep $= \bar{v}A$ in unit time.

If e is the charge on the electron 'n' is the number of electron per unit volume. Then quantity of the charge crossing the given point in the conductor per unit area per unit time is given by

current $I = ne\bar{v}A \quad V = IR = nev_d A . \dfrac{\rho l}{A} = El$

$$\sigma = nev_d/E$$

$\sigma = \dfrac{ne\bar{v}A}{AE} = \dfrac{ne\bar{v}}{E}$

i.e. $\quad \sigma = \dfrac{ne}{E}\dfrac{eE}{m}\tau$

$\sigma = \dfrac{ne^2\tau}{m}$

6. *Discuss the failure of classical free electron theory.*
Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

Specific heat: The molar specific heat of a gas at constant volume is $C_v = \dfrac{3}{2}R$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also. But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was $C_V = 10^{-4}RT$ which is for lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

Temperature dependence of electrical conductivity:
Experimentally, electrical conductivity σ is inversely proportional to the temperature T.

i.e. $\quad\quad\quad\quad\quad\quad \sigma_{exp} \alpha \ 1/T \quad\quad\quad\quad\quad\quad \rightarrow (1)$

According to the assumptions of classical free electron theory

$$\frac{3}{2}KT = \frac{1}{2}mv_{th}^2$$

$$v_{th} = \sqrt{\frac{3KT}{m}}$$

$$ie\ v_{th}\alpha\sqrt{T}$$

The mean collision time 'τ' is inversely proportional to the thermal velocity.

$$\text{i.e. } \tau \ \alpha \ \frac{1}{v_{th}} \qquad \text{or } \tau \ \alpha \ \frac{1}{\sqrt{T}}$$

But $\quad \sigma = \dfrac{ne^2\tau}{m}$

$$\sigma \ \alpha \ \tau$$

or $\quad \sigma \ \alpha \ \dfrac{1}{\sqrt{T}}$ $\qquad\qquad\qquad\qquad \rightarrow(2)$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

Dependence of electrical conductivity on electron concentration:

According to the theory

$\sigma = \dfrac{ne^2\tau}{m}$; where n is the electron concentration, therefore $\sigma \ \alpha \ n$

Consider zinc and cadmium which are divalent metals. Their electrical conductivities are $1.09\times10^{7}/\Omega m$ and $0.15\times10^{7}/\Omega m$. These are much lesser than that of copper and silver. The values of which are $5.88\times10^{7}/\Omega m$ and $6.3\times10^{7}/\Omega m$ respectively. The electron concentrations for zinc and cadmium are $13.1\times10^{28}/m^3$ and $9.28\times10^{28}/m^3$ which are much higher than that for copper and silver, the values of which are $8.45\times10^{28}/m^3$ and $5.85\times10^{28}/m^3$ respectively. Hence the classical free electron theory fails to explain the dependence of σ on electron concentration.

Quantum free electron theory

1. *State the assumptions of quantum free electron theory.*
The assumptions of quantum free electron theory are:

The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.

The following assumptions of classical free electron theory holds good in quantum free electron theory also.
The electrons travel with a constant potential inside the metal but confined within its boundaries.
The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

2. *Explain density of states.*
There are large numbers of allowed energy levels for electrons in solid materials. A group of energy levels close to each other is called as energy band. Each energy band is spread over a few electron-volt energy ranges. In 1mm³ volume of the material, there will be a more than a thousand

billion permitted energy levels in an energy range of few electron-volts. Because of this, the energy values appear to be virtually continuous over a band spread. To represent it technically it is stated as density of energy levels. The dependence of density of energy levels on the energy is denoted by g(E). It is called density of states function. It is the number of allowed energy levels per unit energy interval in the band associated with material of unit volume. In an energy band as E changes g(E) also changes.

Consider an energy band spread in an energy interval between E_1 and E_2. Below E_1 and above E_2 there are energy gaps. g(E) represents the density of states at E. As dE is small, it is assumed that g(E) is constant between E and E+dE. The density of states in range E and

Energy interval in an Energy band
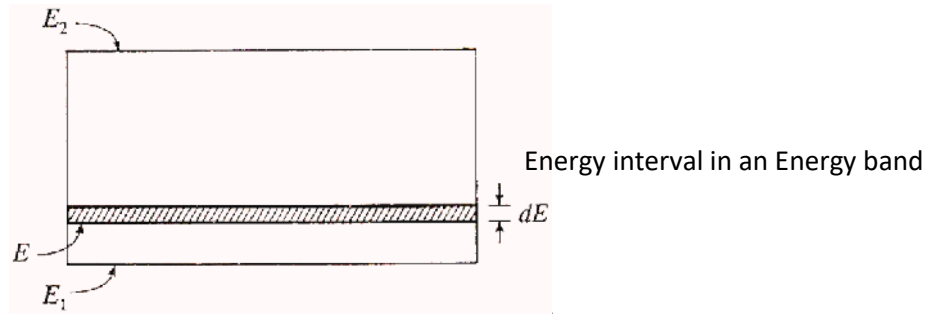
E+dE is denoted by g(E)dE.

i.e.
$$g(E)dE = \left[\frac{8\sqrt{2}\,\pi\,m^{\frac{3}{2}}}{h^3} \right] E^{\frac{1}{2}}\, dE$$

It is clear g(E) is proportional to $\sqrt{E}$ in the interval dE

The graph shows variation of g(E) versus E.

DENSITY OF STATES
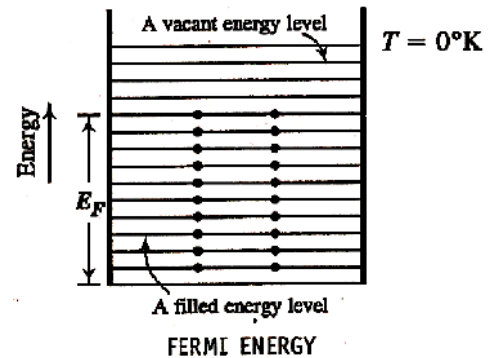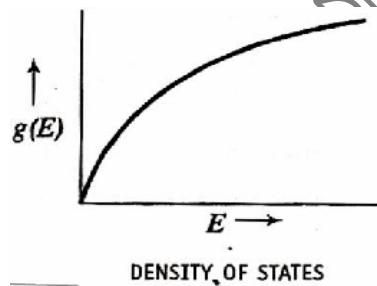
FERMI ENERGY

Explain Fermi energy and Fermi factor. Discuss the variation of Fermi factor with temperature and energy.

Fermi energy: In a metal having N atoms, there are N allowed energy levels in each band. In the energy band the energy levels are separated by energy differences. It is characteristic of the material. According to Pauli's exclusion principle, each energy level can accommodate a maximum of two electrons, one with spin up and the other with spin down. The filling up of energy levels occurs from the lowest level. The next pair of electrons occupies the next energy level and so on till all the electrons in the metal are accommodated. Still number of allowed energy levels, are left vacant. This is the picture when there is no external energy supply for the electrons. The energy of the highest occupied level at absolute zero temperature (0K) is called the Fermi energy and the energy level is called Fermi level. It is denoted by 'E_f'

Fermi factor: The electrons in the energy levels for below Fermi level cannot absorb the energy above absolute zero temperature. At ordinary temperature because there are no vacant energy levels above Fermi level into which electrons could get into after absorbing the thermal energy. Though

the excitations are random, the distributions of electrons in various energy levels will be systematically governed by a statistical function at the steady state.

The probability f(E) that a given energy state with energy E is occupied at a steady temperature T is given by

$$f(E) = \cfrac{1}{e^{\cfrac{(E-E_F)}{kT}} + 1}$$

f(E) is called the fermi factor.

Fermi factor is the probability of occupation of a given energy state for a material in thermal equilibrium.

The dependence of fermi factor on temperature and energy is as shown in the figure.

Following are the different cases.

Probability of occupation for E<E_f at T=0 K:

When T=0K and E<E_f

$$f(E) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

f(E)=1 for E<E_f.

f(E)=1 means the energy level is certainly occupied and E<E_f applies to all energy levels below E_f. Therefore at T=0 all the energy levels below the fermi level are occupied.

VARIATION OF $f(E)$ WITH E
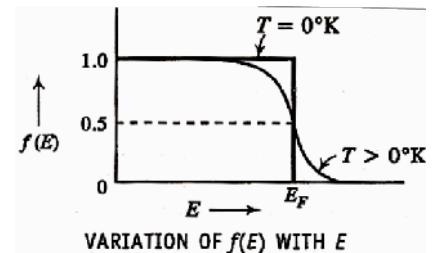
Probability of occupation for E>E_f at T=0 K:
When T=0K and E>E_f
$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0$$

∴ f(E)=0 for E>E_f

∴ At T=0K, all the energy levels above fermi levels are unoccupied. Hence at T=0K the variation of f(E) for different energy values, becomes a step function as shown in the figure.

The probability of occupation at ordinary temperature:
At ordinary temperatures though the value of probability remains 1, for E<E_f it starts reducing from 1 for values of E close to but lesser than E_f as in the figure.
The values of f(E) becomes ½ at E=E_f
This is because for E=E_f
$$e^{(E-E_f)/kT} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{e^{(E-E_f)/kT}+1} = \frac{1}{1+1} = \frac{1}{2}$$

Further for E>E$_f$ the probability value falls off to zero rapidly. Hence, the fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute.

5) *Describe Fermi-Dirac distribution and discuss the same for different temperature conditions.*

Fermi-Dirac distribution deals with the distribution of electrons among the permitted energy levels. The permitted energy levels are the characteristics of the given material. The density of the state function g(E) changes within a band. The number of energy levels in the unit volume of the material in the energy range E & (E+dE) is g(E)dE.

Each electron will have its own energy value which is different from all others except the one with opposite spin. The number of electrons with energy range E & (E+dE) in unit volume is N (E) dE which is the product of the number of energy levels in the same range and the fermi factor.

$\therefore$ N (E) dE = f(E)×g(E)dE.

FERMI-DIRAC DISTRIBUTION AT DIFFERENT TEMPERATURES

But f(E) and g(E) at a temperature T changes only with E. i.e., N(E)dE only at a given temperature change with E.

The plot of N(E)dE vs E represents the actual distribution of electrons among the available states for the material for the temperature. The distribution is known as Fermi–Dirac distribution. Fermi-Dirac distribution represents the detailed distribution of electrons among the various available energy levels of a material under thermal equilibrium conditions.

Fermi-Dirac distribution can be considered in the following three conditions: At T=0K, at T> 0K and T>>0K.

The plot of N(E) vs E for all the three cases is in the fig.

Case(1): T=0K

From the graph at T=0K, N(E) increases with E, upto E=E$_f$. This is due to increase in g(E) with E. Beyond E=E$_f$, we know f(E)=0 for T=0K. Thus N(E)=0 for all the values of E>E$_f$ regardless the value that g(E) possesses in this range. This is represented as a sudden drop of N(E) to zero in the graph at E=E$_f$.

Case(2): T > 0K

At ordinary temperatures, the values of f(E) changes slightly near the fermi energy on the either side of it, as compared to it's value of zero at T=0K. Correspondingly N(E) also undergoes a light variation with small decrease below E$_f$ and small increase above E$_f$ near E$_f$. Because electrons are depleted from energy levels below E$_f$ and populate the electrons above E$_f$ both occurring near E$_f$. But for energy values away from E$_f$ since f(E) remains same for both T=0K and T>0K. N(E) is same i.e., the electrons which possess energy quite lower than E$_f$ are unaffected at ordinary temperature.

Case(3): T>>0K

For very high temperatures of the order of 1000K, f(E) changes from what it was at T=0K even for energy values which are not quite close to E$_F$. This in turn has effect on N(E) with its value undergoing reduction and increase respectively over a large range of energy states below and above E$_F$ respectively.

i.e. at T>>0K the electrons at levels much lower to E_f are elevated by thermal excitation to levels above E_f over a higher energy range. But even at high temperature conditions a significant portion of the graph remains same as that at T=0K. i.e. the electrons at very low energy levels remain undisturbed inspite of high thermal energy input.

Hence the distribution of the electrons in the various energy levels is controlled by the fermi factor. i.e. Fermi factor is also known as Fermi-Dirac distribution function.

6) *Give the theory for calculation of fermi energy at T=0K and T>K.*

Fermi energy 'E_F' at 0K is denoted E_{F0}.

The number of electrons/unit volume which possess energy only in the range E and E+dE is given by N(E)dE

= [number of available states in the energy range E and (E+dE)]×[probability of the occupation of those energy levels by the electron.]

If g(E) is the density of state function, then the number of energy states in the range E and (E+dE) =g(E) and the probability of occupation of any given energy state by the electron is given by the fermi factor f(E).

∴ N(E)dE=g(E)dE×f(E)

The number of electrons/unit volume of the material 'n' can be evaluated by integrating the above expression from E=0 to E=E_{max} , where E_{max} is the maximum energy possessed by the electrons.

$$n = \int_{E=0}^{E_{max}} N(E)dE$$

Or $\quad n = \int_{E=0}^{E_{max}} g(E)f(E)dE$

But f(E)=1 at T=0 K and E<E_f

∴ $\quad\quad n = \int_{E=0}^{E_{max}} g(E)dE \times 1$

Where $g(E)dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}dE$

Where 'm' is the mass of the electron and 'h' is the Planck 's constant.

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_{E=0}^{E_{max}} E^{1/2}dE$$

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \left[\frac{2}{3}E^{3/2}\right]_0^{E_{max}}$$

But at T=0K, the maximum energy that any electron of the material can have is E_{Fo}. Hence $E_{max}= E_{Fo}$.

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\frac{2}{3}(E_{Fo})^{3/2}$$

$$n = \left[\frac{8\times 2^{3/2}m^{3/2}}{h^3}\right]\frac{\pi}{3}(E_{Fo})^{3/2} \quad\quad\quad ($$

∵ $\quad 8^{\frac{3}{2}} = \sqrt{8\times 8\times 8} = \sqrt{8^2}\times\sqrt{8} = 8\times\sqrt{2\times 2\times 2} = 8\times 2^{\frac{3}{2}}$)

$$(E_{Fo})^{3/2} = \left[\frac{h^3}{(8m)^{3/2}}\right]\frac{3n}{\pi}$$

$$E_{Fo} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}$$

$$E_{Fo} = Bn^{2/3}$$

Where $B = \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3}$ is a constant $= 5.85 \times 10^{-38}$J

Fermi energy E_F at any temperature, T in general can be expressed in terms of E_{Fo} through the relation

$$E_F = E_{Fo}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{Fo}}\right)^2\right]$$

Except at extremely high temperature, the second term within the brackets is very small compared to unity. Because $E_F = E_{Fo}$.

Hence at ordinary temperature the values of E_{Fo} can be taken to be equal to E_F.

Describe how quantum free electron theory has been successful in overcoming the failures of classical free electron theory

Quantum free electron theory has successfully explained following observed experimental facts where as the classical free electron theory failed.

Specific heat: According to classical free electron theory all the conduction electron are capable of observing the heat energy has per Maxwell Boltzmann statistics which results in large value of specific heat.

According to quantum free electron theory, it is only those electron that are occupying energy levels close to E_F , which are capable of observing the heat energy to get excited to higher energy levels .

Hence only a small percentage of the conduction electrons are capable of receiving the thermal energy input, thus the specific heat value becomes very small for the metal.

According to quantum free electron theory, it can be shown

$$C_v = \left(\frac{2k}{E_F}\right)RT$$

Considering $E_F = 5$ eV

$$\left(\frac{2k}{E_F}\right) \approx 10^{-4}$$

$\therefore C_v = 10^{-4}$ RT which is close to experimental value.

b) Temperature depends on electrical conductivity.

Electrical conductivity σ is proportional to $\frac{1}{T}$ but not $\frac{1}{\sqrt{T}}$ which is as follows:

Electrical conductivity $\sigma = \frac{n\tau e^2}{m^*}$;where m* is called as effective mass.

According to quantum free electron theory τ is $\tau = \frac{\lambda}{v_F}$

$$\therefore \sigma = \frac{n\lambda e^2}{m^* V_f}.$$

According to quantum free electron theory E_F and V_F are independent of temperature. The dependence of λ & T is as follows

Conduction electrons are scattered by the vibrating ions of the lattice. The vibration occurs such that the displacement of ions takes place equally in all directions. If 'r' is the amplitude of vibrations then the ions can be considered to present effectively a circular cross section of area πr^2 that blocks the path of the electron irrespective of direction of approach. Since the vibrations are larger area of cross section should scatter more number of electrons, it results in a reduction the value of mean free path of the electron.

$$\therefore \lambda \; \alpha \; \frac{1}{\pi r^2}$$

Considering the facts

The energy of vibrating body is proportional to the square of amplitude.
The energy of ions is due to the thermal energy.
The thermal energy is proportional to the temperature 'T'.

We can write $r^2 \; \alpha \; T$

$$\therefore \lambda \; \alpha \; 1/T$$

$$\therefore \sigma \; \alpha \; 1/T$$

Thus $\sigma \; \alpha \; 1/T$ is correctly explained by quantum free electron theory.

Electrical conductivity and electron concentration:

Aluminium and gallium which have three free electrons per atom have lower electrical conductivity than that of copper and silver, which have only one free electron per atom.

As per quantum free electron the electrical conductivity is

$$\sigma = \frac{ne^2}{m^*}\left(\frac{\lambda}{v_f}\right)$$

It is clear that copper and aluminium the value of n for aluminium is 2.13 times higher than that of copper. But the value of λ/v_f for copper is about 3.73 times higher than that of aluminium. Thus the conductivity of copper exceeds that of aluminium.

9) State the main assumptions of quantum free electron theory along with those which are applicable from classical free electron theory also.
Similarities between the two theories:
The valence electrons are treated as though they constitute an ideal gas.
Valence electrons can move freely throughout the body of the solid.
The mutual collisions between the electrons and the force of attraction between the electrons and ions are considered insignificant.
Difference between the two theories:
According to classical free electron theory:
The free electrons which constitute the electron gas can have continuous energy values.
It is possible that many electrons possess same energy.
The pattern of distribution of energy among the free electron obeys Maxwell-Boltzmann statics.

According to quantum free electron theory:

The energy values of the free electrons are discontinuous because of which their energy values are discrete. The free electrons obey the Pauli's exclusion principle. Hence no two electrons can possess same energy. The distribution of energy among the free electrons is according to Fermi-Dirac statistics, which imposes a severe restriction on the possible ways in which the electrons absorb energy from an external source.

Physics of Semiconductor: Fermi level in intrinsic semiconductors, Expression for concentration of electrons in conduction band, Hole concentration in valance band (only mention the expression), Conductivity of semiconductors(derivation), Hall effect, Expression for Hall coefficient(derivation)

SEMICONDUCTORS
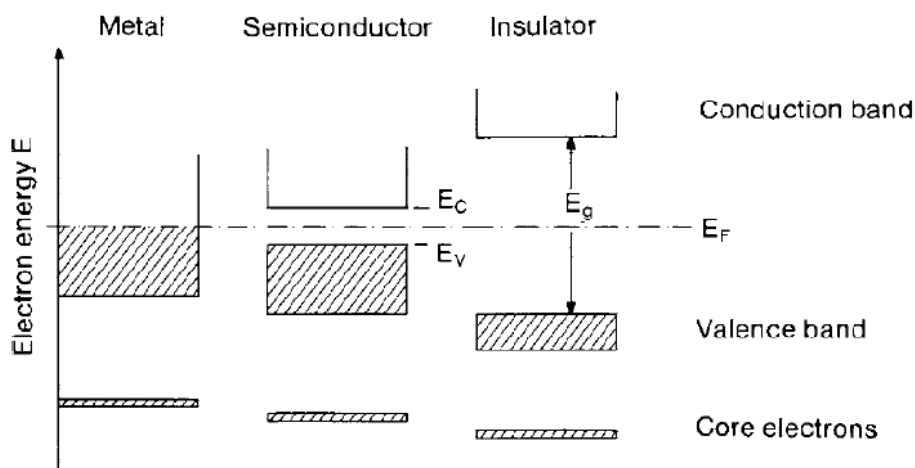
Energy level diagram for a metal, semiconductor and an insulator

	$E_g\ (T=0\ \text{K})$ [eV]	$E_g\ (T=300\ \text{K})$ [eV]
Si	1.17	1.12
Ge	0.75	0.67

Energy gap in Germanium and Silicon

Intrinsic semiconductors:Type equation here.

	E_g [eV]	n_i [cm^{-3}]
Ge	0.67	2.4×10^{13}
Si	1.1	1.5×10^{10}
GaAs	1.43	5×10^{7}

Carrier concentration at 300K

Conductivity of n-type semiconductors:

Here the electron concentration is far greater than the hole concentration.

$$\mu = \frac{v_d}{E} = \frac{eE\tau}{mE} = \frac{e\tau}{m}$$

$$\sigma = \frac{ne\ e\tau}{m} = ne\mu$$

$$\therefore \sigma = n_e e \mu_e + n_{hole} e \mu_{hole}$$

$$As\ n_e > n_p$$

Department of Physics

$$\sigma = n_e e \mu_e$$

Conductivity of p-type semiconductors:

Here the hole concentration is far greater than the electron concentration.

$$\therefore \sigma = n_e e \mu_e + n_{hole} e \mu_{hole}$$

$$\text{As } n_p > n_e$$

$$\sigma_{int} = n_h e \mu_h$$

Conductivity of Intrinsic semiconductors:

Current density $J = n\,e\,V_d$

For a semiconductor, $J = n_e\,e\,V_d(e) + n_h\,e\,V_d(h)$ ....(1)

But drift velocity $V_d = \mu E = \mu.J/\sigma$

Using (1), $\sigma = n_e\,e\,\mu_e + n_h\,e\,\mu_h$

In an intrinsic semiconductor, number of holes is equal to number of electrons.

, $\sigma_{int} = n_e e[\mu_e + \mu_{hole}]$

n_e is the electron concentration

n_p is the hole concentration

μ_e is the mobility of electrons

μ_h is the mobility of holes

ELECTRON DENSITY IN CONDUCTION BAND

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi n_e^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band is given by

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$
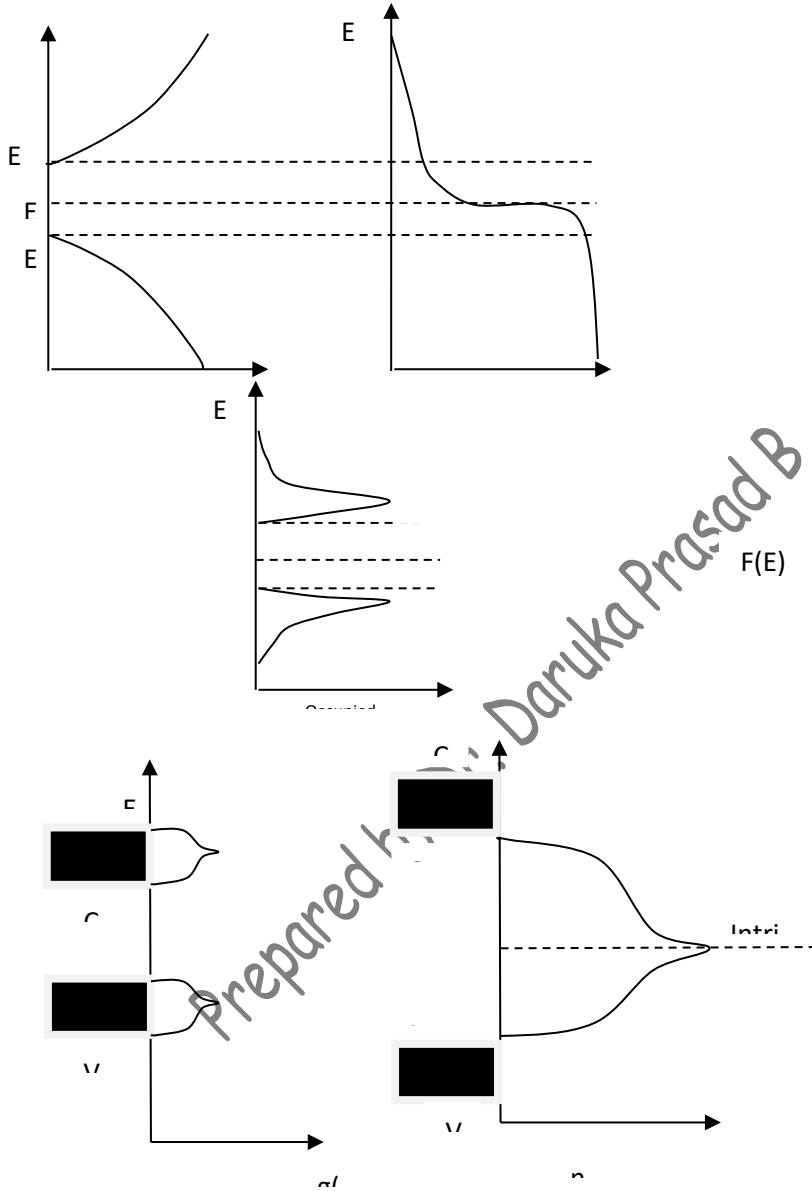
Expression for Fermi Level in Intrinsic Semiconductor

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$
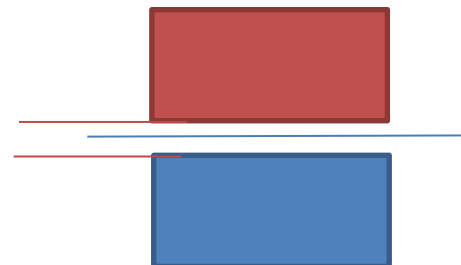
Hall effect: When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.
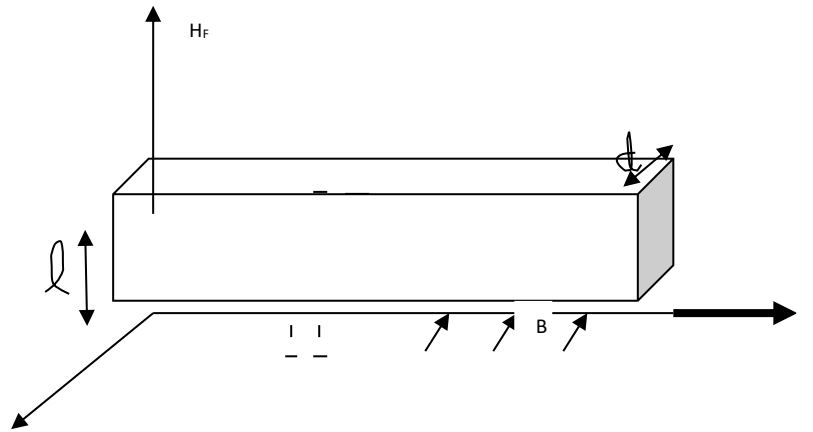
Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

Expression for electron concentration:

At equilibrium, Lorentz force is equal to force due to electrons

$$BeV_H = -e\, E_H$$

$$E_H = BV_H$$

But $E_H = \dfrac{V_H}{l}$

$$V_H = l\, E_H = B\, l\, V$$

$$\text{Current density } J = n_e ev = \frac{I}{\text{Area}} = \frac{I}{ld} \quad \text{..............(1)}$$

Current density J

$$v = \frac{I}{n_e ld}$$

Hence $V_H = \dfrac{BLI}{n_e ld}$ ....(2)

Electron concentration $n_e = \dfrac{BI}{V_H d}$

Expression for Hall coefficient (R_H)

Hall field is directly proportional to current density (J) and Magnetic field(B).

$E_H \, \alpha \, B$

$E_H \, \alpha \, J$

$E_H = R_H \, J \, B$ where R_H is a constant known as Hall coefficient.

$$E_H = R_H \, J \, B$$

$$R_H = \frac{E_H}{JB} = \frac{1}{n_e} \qquad \text{(From (1) and (2))}$$

Expression for mobility of charge carrier

Mobility $\quad \mu = \dfrac{v}{E} \quad$ ....(1)

Current density $J = \sigma E = n_e e \mu E = n_e ev$ ......(2)

Simplifying (1) and (2)

$$\mu = \sigma R_H$$

Hall effect can be used to

1. Determine the type of semiconductor

2. Calculate carrier concentration, mobility

3. To calculate Magnetic flux density B

4. To determine power in a electromagnetic waves

Dielectric materials: polar and non-polar dielectrics, internal fields in a solid, Clausius-Mossotti equation(Derivation), mention of solid, liquid and gaseous dielectrics with one example each. Application of dielectrics in transformers.

DIELECTRIC PROPERTIES OF MATERIALS

Dielectrics are insulators, they do not have free electrons, and they do not conduct electricity. They affect the electric field in which they are placed.

A pair of equal and opposite point charges separated by a small distance is called electric dipole. The product of the magnitude of one of the charges and the distance between them is called the dipole moment.

$$\mu = q.l$$

Polarization: When an electric field is applied to dielectric material, there is displacement of charged particles leading to formation of dipoles and hence dipole moment which is called polarization of dielectric.

Dielectrics are of two types: 1) Polar dielectrics 2) Non polar dielectrics

1) In a polar dielectric molecule the centers of positive and negative charge distributions are separated by a small distance. They act like tiny poles and posses permanent electric dipole moment.

   In the absence of external field, the dipoles are oriented randomly, it results in a net zero dipole moment for the material.

2) In non polar dielectric molecule the centers of positive and negative charge distribution coincide. It has no permanent dipole moment. In the presence of an external field the charge distribution are separated by a small distance and acquire dipole moment. It is the induced dipole moment.

   The relation between electric intensity E and the flux density D for an isotropic material is given by

$$D = \epsilon_o \epsilon_r E$$

Where $\epsilon_0 = 8.854 \times 10^{-12}$ F/m dielectric constant of air or vacuum.

ϵ_r is the relative permittivity of the materials.
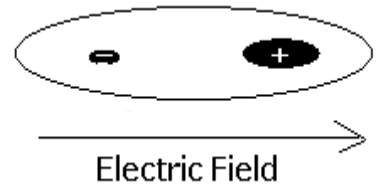
Electric Field

Electric Polarization and Dielectric susceptibility χ

Consider a dielectric material of area A subjected to an external electric field E. 't' is the thickness of the slab +q & -q be the induced charges.

The total dipole moment of the material = (charge) x (distance of separation)

$$\mu = q \times t$$

The dipole moment per unit volume is called the polarization P.

i.e., $$P = \frac{Total\ dipole\ moment\ of\ the\ material}{volume\ of\ the\ material} = \frac{qt}{tA} = \frac{q}{A}\ cm^{-2}$$

Thus magnitude of polarization is equal to the induced charge density. But polarization P is directly proportional to the applied field E.

i.e., P α E

$$P = \epsilon_o\ \chi\ E$$

where χ is called dielectric susceptibility.

Relation between polarization P and Dielectric constant ϵ_r:

Consider a dielectric slab placed between the two plates and subjected to external electric field E_0. σ be the charge per unit area of the plates. By Gauss theorem $E_o = \frac{\sigma}{\epsilon_0}$ --------------------(1)

Because of polarization of the slab, a field E' is established within the slab. This field is opposite to that of E_o.

The resultant field $E = E_o - E'$ -------------- (2)

If σ_p is the charge/unit area on the slab surface, then similar to eqn. (1)

$$E' = \frac{\sigma_p}{\epsilon_0}$$ ----------------------------(3)

From equations (1), (2) & (3)

$$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$$

$$\epsilon_o E = \sigma - \sigma_p$$ --------------------(4)

i.e., $\epsilon_o E = D - P$ [$\because$ P = charge/unit area; P = σ_P ; D = σ By Gauss law]

$$D = \epsilon_o E + P$$ -------------------(5)

But   D=$\epsilon_o \epsilon_r$E

$\therefore \epsilon_o \epsilon_r$E = ϵ_0E+P

$$\vec{P} = \varepsilon_0 \left(\varepsilon_r - 1 \right) \vec{E}$$

$$\boxed{P = \epsilon_o \chi \, E}$$

where $\chi = (\epsilon_r - 1)$ is dielectric susceptibility of the material.

Polarizability (α)

The dipole moment μ acquired by the dielectric atom or molecule is proportional to the applied electric field E

i.e., $\mu \; \alpha \; E$

i.e., $\mu = \alpha \; E$

where α is the polarizability of the atom. Its unit is Fm^2.

TYPES OF POLARIZATION: There are four different types of polarization. They are,

1) Electronic Polarization,
2) Ionic Polarization
3) Oriental polarization and
4) Space charge polarization.
1) _Electronic polarization_ : There is displacement of positive and negative charges due to applied external electric field. This leads to development of dipole moment. Thus material gets polarized.

The electronic polarization $\alpha_e = \dfrac{\varepsilon_0 \left(\varepsilon_r - 1 \right)}{N}$

Where N is the number of atoms per unit volume.

Electronic polarization
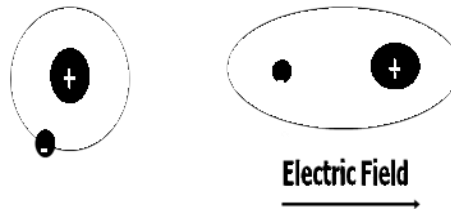
Electric Field

2) **_Ionic Polarization :_** There is displacement of adjacent opposite ions due to applied external electric field. Depending on the location of ions there is increase or decrease in displacement of ions. This leads to development of dipole moment of the material.
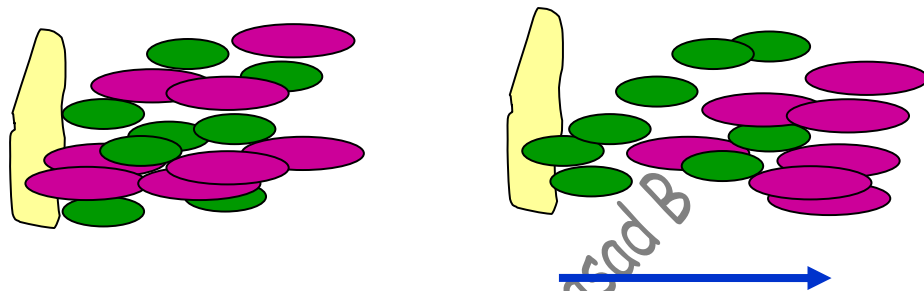

3) **_Orientation polarization :_** In the absence of the external field, the dipoles are oriented randomly, the net dipole moment is zero. In the presence of the external field each of the dipoles undergo rotation so as to reorient in the direction of the field. Thus material develops electrical polarization. It is the temperature dependant and decreases with increase of temperature.

It is given by $\alpha_0 \dfrac{\mu^2}{3KT}$

μ-permanent dipole moment,
k-Boltzmann constant, T- Temperature.

Electric field

+e

-e
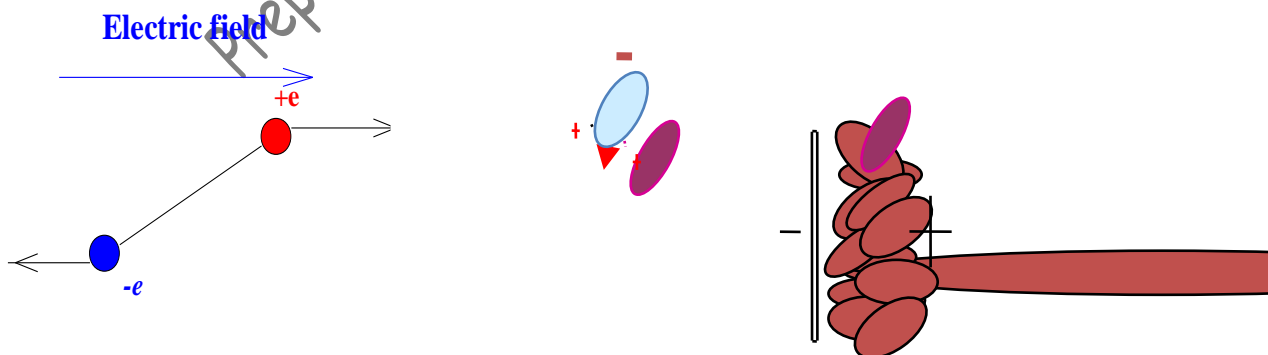
4) **_Space charge polarization:_** It occurs in multiphase dielectric materials where there is change of resistivity between different phases. At high temperatures when the material is subjected to electric field charges are settled at the interface due to sudden drop of conductivity across the boundary. Opposite nature of charges are settled at opposite parts in the low resistivity phase. Thus the material acquires dipole moment in the low resistivity phase. Space charge polarization is negligible in most dielectrics.
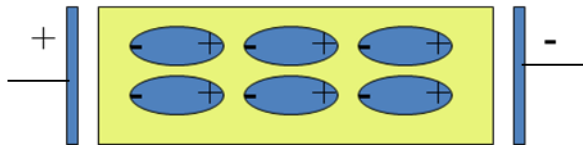
Temperature dependence of polarization mechanism

The distribution of electrons in the constituent molecules is affected by the increase in the temperature. Thus there is no influence on the electronic and ionic polarization mechanisms. But the increase in temperature changes the dipole orientation established by the applied field. This changes the orientation polarization. The orientation polarization is inversely proportional to the temperature. The thermal energy support in movement by diffusion which intern aids the molecules to align in the field direction. Thus increase in temperature supports space charge polarization and orientation polarization.

Expression for the internal field in the case of liquids and solids:
(One dimensional)

Internal or local field is the resultant of the applied field and field due to all the surrounding dipoles on an atom of a solid or a liquid dielectric material.

Consider an array of equivalent atomic dipoles arranged parallel to the direction of the uniform field E. Let 'd' be the inter-atomic distance and μ be the dipole moment of each dipole.

The total field E_i at x is the sum of the applied E and the field due to all the dipoles E'

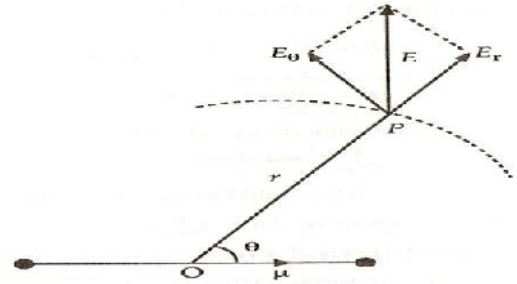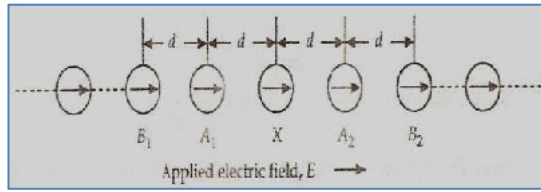$$E_i = E + E' \qquad (1)$$

E' is found as follows

The components of the electric field at P due to a dipole in polar form is given by

$$E_r = \frac{\mu \cos\theta}{2\pi\varepsilon_0 r^3}$$

and $\quad E_\theta = \frac{\mu \sin\theta}{4\pi\varepsilon_0 r^3}$

where μ is the dipole moment

Since d=r & θ=0⁰

$$E_r = \frac{\mu}{2\pi\varepsilon_0 d^3} \quad \& \ E_\theta = 0$$

∴ Field at x due to A_1 = $E_r + E_\theta$ = $\dfrac{\mu}{2\pi\varepsilon_0 d^3}$

Similarly Field at X due to A_2= $\dfrac{\mu}{2\pi\varepsilon_0 d^3}$

∴ The net field at X due to A_1 & A_2 is E_1= $2\dfrac{\mu}{2\pi\varepsilon_0 d^3} = \dfrac{\mu}{\pi\varepsilon_0 d^3}$

The net field at X due to B_1 & B_2 is E_2= $\dfrac{\mu}{2\pi\varepsilon_0 (2d)^3}$

The net field at X due to C_1 & C_2 is E_3= $\dfrac{\mu}{2\pi\varepsilon_0 (3d)^3}$

∴ The total field E' at X due to all the dipoles is

E'= $E_1 + E_2 + E_3$+..............................

$$= \frac{\mu}{\pi\varepsilon_0 d^3} + \frac{\mu}{2\pi\varepsilon_0 (2d)^3} + \frac{\mu}{2\pi\varepsilon_0 (3d)^3} + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \frac{\mu}{\pi\varepsilon_0 d^3}\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \ldots\ldots\ldots\ldots\right)$$

$$= \frac{\mu}{\pi\varepsilon_0 d^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \ ;$$

$$= \frac{1.2\mu}{\pi\varepsilon_0 d^3} \qquad\qquad \because \sum_{n=1}^{\infty} \frac{1}{n^3} = 1.2$$

The internal field $\qquad E_i = E + \dfrac{1.2\mu}{\pi\varepsilon_0 d^3}$ (4)

$$E_i = E + \frac{1.2\alpha_e E}{\pi\varepsilon_0 d^3}$$

Where $\mu = \alpha_e E$, α_e is the electric polarizability of dipoles.
The equation for internal field in three dimension is given

$E_i = E + \left(\dfrac{\gamma}{\varepsilon_0}\right) p$, where p is the dipole moment per unit volume for the material. γ is internal

field constant.

For cubic lattice internal field is called Lorentz field.

$$E_{Lorentz} = E + \left(\frac{p}{3\varepsilon_0}\right) \qquad\qquad where\ \gamma = \frac{1}{3},$$

CLAUSIUS – MOSSOTTI RELATION:

Consider a dielectric material of dielectric constant ε_r.
The dipole moment / unit volume= Nμ
Where N is the number of atoms per unit volume, μ is the dipole moment of each atom.
The dipole moment / unit volume = N $\alpha_e E_i$

$$E_i = \frac{p}{N\alpha_e} \tag{1}$$

But $p = \epsilon_0 (\epsilon_r - 1)E$ where E is the applied field

$$\therefore E = \frac{p}{\epsilon_0 (\epsilon_r - 1)} \tag{2}$$

We have

$$E_i = E + \gamma\frac{p}{\epsilon_0} \tag{3}$$

where γ is the internal field constant

From Equations (1), (2) *and* (3)

$$\frac{p}{N\alpha_e} = \frac{p}{\epsilon_0 (\epsilon_r - 1)} + \gamma \frac{p}{\epsilon_0}$$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_0}\left[\frac{1}{(\epsilon_r - 1)} + \gamma\right]$$

Taking internal field in the material to be Lorentz field $\gamma = \frac{1}{3}$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_0}\left[\frac{1}{(\epsilon_r - 1)} + \frac{1}{3}\right] = \frac{1}{\epsilon_0}\left[\frac{3 + \epsilon_r - 1}{(\epsilon_r - 1)3}\right]$$

$$\frac{\epsilon_0}{N\alpha_e} = \left[\frac{\epsilon_r + 2}{(\epsilon_r - 1)3}\right]$$

$$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \left[\frac{N\alpha_e}{3\epsilon_0}\right]$$

This is Clausius- Mossotti equation.

Dielectric loses:- It is the loss of energy in the form of heat due to internal friction that is developed as a consequence of switching action of dipoles under certain a.c. conditions.

Dipolar relaxation:- It is the time required for the dipole to reach the equilibrium orientation from the disturbed position in an alternating field condition. The reciprocal of relaxation time is the relaxation frequency.

Frequency dependence of Dielectric constant.

The dielectric constant ε_r of a dielectric material changes with the frequency of the applied voltage. If the frequency is low the polarization is following the variation of the field without any lag. As the frequency increases the heavy positive and negative ions cannot follow the field variations. ε_r becomes a complex quantity. It is denoted as ε_r^* given by

$\varepsilon_r^* = \varepsilon_r' - j\varepsilon_r''$, where ε_r' and ε_r'' are real and imaginary parts of ε_r^* .
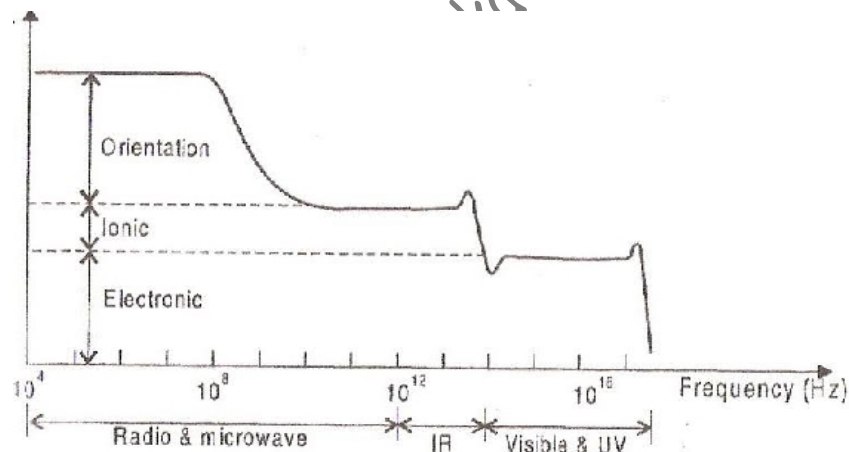
All the four different polarization mechanisms respond differently at different frequencies under alternating field conditions, because relaxation frequencies of different polarization processes are different.

$$\tau_e < \tau_i < \tau_0$$

As the frequency of the applied a.c. is increased, different polarization mechanisms disappears in the order, orientation, ionic and electronic.

The peaks in the variation of ε_r'' over frequency regions corresponding to the decrements in ε_r' indicates the losses that the material suffer over those frequencies. It can be shown that

$$\tan \delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$; where δ is phase angle. Large value of tan δ refers to higher dielectric loss. It is called *tangent loss.*

Applications of dielectric materials

Dielectric materials are used in capacitors to increase charge storage capacity. Quartz, Lead Zirconate titanate, Rochelle salt, Barium titanate and poly vinylidene fluoride are piezoelectric materials. Quartz is piezoelectric but not ferroelectric. It is in the form of SiO_2. Piezoelectric crystals used in Electronics industry in frequency control of oscillators. A properly cut piezoelectric crystal is placed in between the plates of a capacitor of a circuit whose frequency is same as the natural frequency of mechanical vibration of the crystal. The circuit acts as a tuned circuit of very high Q-value and possesses excellent frequency stability. They are also used as electro-acoustic transducers (to convert electrical energy into mechanical and vice versa). Transducers are used in ultrasonic's for Sound Navigation and Ranging (SONAR), in ultrasound imaging of human body, non-destructive testing of materials, measurement of velocity of ultrasound in solids and liquids.

Lead Zirconate titanate (Pb Ti_{1-x} Zr_x O_3) or (PZT) are used in accelerometers, earphones etc., PZT piezoelectric crystals are used in gas lighters, car ignition.

Rochelle salt (Na KC_4 H_4O_6 $4H_2O$) is both piezoelectric and ferroelectric, it is hygroscopic and could be used in the range of temperature of 18^0 to 24^0c. It is highly sensitive.

Barium titanate (Ba Ti O_3) is less sensitive than Rochelle salt. It has an advantage of serving over a wide range of temperature. It can withstand atmospheric corrosion. It is used in accelerometers. Polyvinylidene fluoride (PVDF) is inexpensive.

Solid Dielectrics

- Jacketing Materials
- Moulding Materials
- Filling Materials

- Moulding Materials : These are used for mechanically rigid forms of insulation, for example, insulators, bushings and so on e.g. are ceramics, glass (toughened glass), fiberglass reinforced plastics and epoxy - resins .
- Jacketing Materials : Jacketing on a conductor for insulation. Polymers have been found suitable for providing extruded insulating jackets to the conductors. For example, polyethylene (PE), polyvinylchloride (PVC), natural and synthetic (ethylene propylene) rubber are extruded on the conductor in power cables. Polypropylene and paper are used in capacitors and transformers. Mica and fiberglass based polypropylene tapes are used in electrical machines.
- Beside oils, wax - based draining and non - draining impregnating compounds of different types are used to impregnate paper used in power cables, transformers, capacitors, and instrument transformers. Insulating Mechanical Support : In the form of plates, pipes and ledges, insulating supports are required in transformers, circuit breakers

and isolators. The products, such as pressboards, hard paper (thin paper laminates), wood (yellow teak) are used in transformers and Bakelite..

- Unlike gaseous and liquid dielectrics, any damage caused to solid dielectrics due to excessive electrical, thermal or mechanical stresses is often irreversible .
- Their thermal and mechanical properties play a very sensitive role since these considerably affect the electrical properties .
- Solid Dielectrics are more exposed to atmosphere, hazards of weather (rain, storm, hail, ice deposits etc.), ultra violet radiation from the sun and pollution (dust, salts etc.)

Liquid Dielectrics

- Insulating oils are used in power and instrument transformers, power cables, circuit breakers, power capacitors, and so on. Liquid dielectrics perform a number of functions simultaneously, namely- insulation between the parts carrying voltage and the grounded container, as in transformers
- impregnation of insulation provided in thin layers of paper or other materials, as in transformers, cables and capacitors, where oils or impregnating compounds are used
- cooling action by convection in transformers and oil fi lled cables through circulation
- filling up of the voids to form an electrically stronger integral part of a composite dielectric
- arc extinction in circuit breakers
- possess a very high electric strength and their viscosity and permittivity vary in a wide range.

CLASSIFICATION OF LIQUID DIELECTRICS

- Organic and Inorganic.
- Organic dielectrics are basically chemical compounds containing carbon. Among the main natural insulating materials of this type are petroleum products and mineral oils, insulating materials are asphalt, vegetable oils, wax, natural resins, wood.A large number of synthetic organic insulating materials are also produced. These are nothing but substitutes of hydrocarbons in gaseous or liquid forms. In gaseous forms are fluorinated and chlorinated carbon compounds. Their liquid forms are chlorinated diphenyles, besides some nonchlorinated synthetic hydrocarbons. The chlorodiphenyles, although possessing some special properties, are not widely used because they are unsafe for humans and very costly.
- Polyisobutylene offers better dielectric and thermal properties than mineral oils for its application in power cables and capacitors, but it is many times more expensive.
- Silicon oils are top grade, halogen free synthetic insulating liquids. They have excellent stable properties, but because of being costly, have so far found limited application for special purposes in power apparatus.
- Among inorganic liquid insulating materials, highly purifi ed water, liquid nitrogen, oxygen, argon, sulphurhexafl uoride, helium etc. have been investigated for possible use as dielectrics.
- Liquefied gases, having high electric strength, are more frequently used in cryogenic applications. Water and water mixtures are being actively investigated for use as

dielectrics in pulse power capacitors and pulsed power modulators, and so on, because of their high relative permittivity, low cost, easy handling and disposal.

Gaseous Dielectrics

By applying a sensible electrical field, the **dielectric gases** can be polarised. Vacuum, Solids, Liquids and Gases can be a dielectric material. A **dielectric gas** is also called as an insulating gas. It is a dielectric material in gaseous state which can prevent electrical discharge. Dry air, Sulphur hexafluoride (SF_6) etc are the examples of gaseous dielectric materials. Gaseous dielectrics are not practically free of electrically charged particles. When electric field is applied to a gas, the free electrons are formed. A few gases such as SF_6 are strongly attached (the electrons are powerfully attached to the molecule), some are weakly attached for e.g., oxygen and some are not at all attached for e.g. N_2. **Examples of dielectric gases** are Ammonia, Air, Carbon dioxide, Sulphur hexafluoride (SF_6), Carbon Monoxide, Nitrogen, Hydrogen etc. The moisture content in dielectric gases may alter the properties to be a good dielectric.

Breakdown in Gases

When subjected to high voltages, gases undergo ionization producing free electrons and begin to conduct.

Properties of Dielectric Gases

The preferred properties of an excellent gaseous dielectric material are as follows

- Utmost dielectric strength.
- Fine heat transfer.
- Incombustible.
- Chemical idleness against the construction material used.
- Inertness.
- Environmentally nonpoisonous.
- Small temperature of condensation.
- High thermal constancy.
- Acquirable at low cost

Application of Dielectric Gases

It is used in Transformer, Radar waveguides, Circuit Breakers, Switchgears, High Voltage Switching, Coolants. They are usually used in high voltage application.

Applications of Dielectrics in transformers

In the case of insulating oil, the ability of the oil (acting as a dielectric) to oppose the flow of-a current is determined by tests which show its dielectric strength. The specific resistance that a dielectric offers under moderate conditions of voltage. The ability to prevent arcing between two electrodes at high electric potentials, or to prevent arcing between the electric source and ground. Petroleum oil is an excellent dielectric and it is used extensively as a hydraulic medium for equipment used around electrical apparatus, such as in servicing high voltage electric equipment, lines, or transmission media. Oil is also used in switchgears, transformers and other such electrical equipment designed to be insulated with a liquid. The dielectric strength of an insulating dielectric-transformer oil is evaluated in terms of its breakdown voltage under a standard set of conditions.

In liquid filled transformers, dielectric fluid is used to cool the windings and provide optimal performance. From the bottom of the tank where the dielectric fluid is at it's lowest or "bottom" temperature, the fluid flows vertically up the winding ducts and is heated by the windings. At the top of the tank, where the fluid is at its highest or "top oil temperature", it exits the main tank and enters a series of radiators or cooling fins. It then flows downward through the radiators, where it is cooled, and reenters the main tank at the bottom. In self cooled transformers this cycle is governed naturally by convection. Natural convection can also be assisted by a series of fans directing air against the radiators increasing the rate of heat transfer and subsequent rate of cooling in the windings. In some large power transformers it is also possible to have a level of forced oil circulation where a pump assists in the circulation of the fluid. This generally provides a lower top oil temperature and more uniform temperatures within the windings. Mixtures of polychlorinated biphenyls (PCBs) used as a transformer dielectric fluid because of their non-flammable nature and chemical stability.